HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics I

Tutorial 4

Please prepare the following questions for discussion in the week beginning Monday 5 February.

1. A whole life policy with sum assured £100,000 is issued to a life aged x. The sum assured is paid at the end of year of death while premiums are paid annually in advance. Let K_x denote the curtate future lifetime of x and L_0 denote the present value of the future loss on the contract, using the premium basis.

   Derive an expression for the standard deviation of L_0.

2. (a) Explain briefly why a life office holds reserves in respect of its life assurance policies.
   (b) A certain life office sells without profits term assurances. It calculates policy values for these policies by the net premium method, using the basis AM92 ultimate, 4% interest. Calculate the policy value for a policy with original term 25 years, effected by level annual premiums 15 years ago by a life then aged exactly 30 for a sum assured of £100,000 payable at the end of year of death. Assume that the policy is still in force and that the annual premium now due is unpaid.
   (c) At the date of issue of the above policy another life office issued a contract providing identical benefits to the same life. This policy is subject to premiums of £200 per annum payable annually in advance, and the office calculates policy values using the prospective gross premium method, on the following basis:

   mortality: AM92 ultimate
   interest: 4% per annum
   future expenses: 10% of office premiums.

   Calculate the policy value for this contract, assuming that the policy is still in force and that the annual premium now due is unpaid.

3. A life office sells non-profit endowment assurances. It calculates net premium policy values assuming the mortality of the AM92 ultimate table. Calculate ${}_5V_{30:\overline{25}|}$, ${}_{10}V_{30:\overline{25}|}$, ${}_{15}V_{30:\overline{25}|}$, and ${}_{20}V_{30:\overline{25}|}$, at 4% p.a. interest. Do you expect the policy values to be higher or lower if you use an interest rate of 5% p.a.

4. An endowment assurance with term 25 years was sold to a life aged 40, 3 years ago. The sum assured is £60,000 payable at the end of year of death and premiums are

payable annually in advance. Bonuses are supercompound (two-tier) of 3% on the sum assured and 4.5% on existing bonuses. What is the policy value (just before payment of the 4th premium)?

Basis:

Method: net premium policy valuation.

Mortality: AM92 select.

Interest: 4% per annum.

5. 15 years ago a life office sold a with-profits whole life policy to a person then aged 40. The sum assured was £20,000 payable at the end of year of death and a premium of £350 was payable annually in advance. Compound reversionary bonus accrues at the start of each year. the bonus added to date is £13,000. The office values the policy on the following gross premium basis.

   Mortality: A1967–70 select.

   Interest: 6% per annum.

   Future bonuses: 2.913% p.a. (compound).

   Expenses: Nil.

   Calculate the policy value just before the 16th premium is paid.

6. A life office issues a large number of 3-year non-profit endowment assurances policies to males aged exactly 62. The sum assured under each policy is £2,000 payable at the date of maturity or at the end of year of earlier death. Under each policy, level annual premiums are payable in advance throughout the term of the policy.

   The premium and reserving bases are as follows.

   Premium basis:

   Mortality: A1967-70 select. Interest: 6% per annum. Expenses: Initial £150

   Renewal £20 at the start of the second and third policy years.

   Reserving basis: Net premium method

   Mortality: A1967-70 ultimate Interest: 3% per annum.

   (a) Show that the annual premium is £665.07.
   (b) Calculate the policy values per policy at durations 0, 1, and 2 years respectively.

7. State and prove the recursive relationship between policy values of (i) a pure endowment with level annual premiums; (ii) a level annuity payable annually in advance for life.