HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics I

Tutorial 3

1. Write down the first two steps of an Euler scheme with step size h for the numerical solution of the differential equation:

$$\frac{d}{dt}\,{}_tq_x = \mu_{x+t}\,(1 - {}_tq_x).$$

2. (a) Tutorial 2, Q.6 (a), asked you to derive Thiele's differential equation for a temporary annuity. Using your answer to that question, write down the first two steps of an Euler scheme with step-size h for solving this equation numerically.

   (b) Tutorial 2, Q.6 (b), asked you to derive Thiele's differential equation for a pure endowment. Using your answer to that question, write down the first two steps of an Euler scheme with step-size h for solving this equation numerically.

3. (Excel exercise): Suppose the force of mortality is given by the Gompertz function:

$$\mu_x = 0.0004 \times 1.09^x.$$

   (a) Using an Euler scheme with step size $h = 0.01$ years to solve the Kolmogorov equation, find approximate values of ${}_tp_{20}$ for $0 \le t \le 10$. How does the approximate value of ${}_{10}p_{20}$ compare with the correct value?

   (b) A person age 20, subject to the force of mortality above, buys a 10-year temporary assurance contract, with sum assured £1 payable immediately on death. The premium rate is payable continuously at rate £0.003 per annum. The force of interest is 0.05 per annum. Using an Euler scheme with step size $h = 0.01$ years to solve Thiele's differential equation, show that the policy value at outset, $V(0)$ is approximately £0.0033.

   (c) Explain why, in this case, $V(0) \neq 0$.

   (d) By trying different values for the annual rate of premium, find the rate of premium that results in $V(0) = 0$.

4. (Excel exercise): The Excel file `amf92_mu.xls` contains values of μ_x for the AM92 ultimate and AF92 ultimate life tables, at intervals of 1 year, 0.1 year and 0.01 year. (The last two have been found approximately by linear interpolation between the values at integer ages.) The file can be downloaded from the course web page:

   `www.ma.hw.ac.uk/~andrea/f79af`.

(a) Using Euler schemes with step sizes 1 year, 0.1 year and 0.01 year to solve the Kolmogorov equation, find approximate values of ${}_5p_{30}, {}_{10}p_{30}$ and ${}_{15}p_{30}$. How accurate are these, compared with the correct values found from the Yellow Tables?

(b) Suppose a man age 30 buys the endowment assurance described in Tutorial 2, Q.7, with term 15 years. The life office uses the same basis for calculating premiums and policy values, namely a force of interest of 0.04 per annum and the AM92 ultimate life table. Find the rate of premium for which $V(0) = 0$ by solving Thiele's differential equation with step size 1 year, 0.1 year and 0.01 year.