HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics II

Tutorial 2

Please prepare the following questions for discussion in the week commencing Monday 22 January 2007.

1. Calculate the following assuming the mortality of the AM92 table, and describe each expression in words:

   (a) ${}_{10}p_{30:40}$

   (b) $q_{30:40}$

   (c) $\mu_{40:50}$

   (d) ${}_{10}p_{[30]:[40]}$

   (e) $q_{[30]:[40]}$

   (f) $\mu_{[40]:[50]}$

   (g) $\mu_{[40]+1:[60]+1}$

   (h) ${}_{3}\big|q_{[30]+1:[40]+1}$

2. Let T_x and T_y be the independent random future lifetimes of two lives age x and y, and define $T_{\min} = \min[T_x, T_y]$ and $T_{\max} = \max[T_x, T_y]$.

   (a) Derive an expression for the density of $T_{\max}$.

   (b) Define $\mathring{e}_{xy} = \mathrm{E}[T_{\min}]$. Show that:

   $$\mathring{e}_{xy} = \int_0^\infty {}_tp_{xy}dt.$$

   (c) Show that $\mathrm{Cov}\,(T_{\min}, T_{\max}) = \left(\mathring{e}_x - \mathring{e}_{xy}\right)\left(\mathring{e}_y - \mathring{e}_{xy}\right)$

   (d) Further let K_{min} be the integer part of $T_{\min}$ and define $e_{xy} = \mathrm{E}[K_{\min}]$. Show that

      i. $e_{xy} = \sum_{t=1}^{\infty} {}_tp_{xy}$. and

      ii. $\mathring{e}_{xy} \approx e_{xy} + \frac{1}{2}$.

   (e) Derive an expression for the 'force of mortality' associated with $T_{max} = \max[T_x, T_y]$, denoted $\mu_{\overline{x+t:y+t}}$. What is its value at $t = 0$? Explain this result.

3. Given that $l_{xy} = 10,000$, $l_{x+10:y} = 9,600$, and $l_{x:y+10} = 9,200$ calculate the probability that, of the two independent lives aged x and y, exactly one will survive for 10 years.

4. Show that:

   (a) $\ddot{a}_{xy} = \sum_{k=0}^{\infty} v^k {}_kp_{xy}$.

   (b) $\ddot{a}_{xy:\overline{n}|} = \ddot{a}_{x:\overline{n}|} + \ddot{a}_{y:\overline{n}|} - \ddot{a}_{\overline{xy}:\overline{n}|}$.

   (c) $A_{\overline{xy}} = A_x + A_y - A_{xy}$.

5. Derive an expression for the variance of the random variable $v^{K_{max}+1}$.

6. For a male aged 70 exact and a female aged 67 exact, who are subject to the mortality of the PMA92 and PFA92 tables respectively, with interest of 4% per annum, find:

   (a) $\ddot{a}_{70:67}$

   (b) $\ddot{a}^{(12)}_{70:67}$

   (c) $\ddot{a}_{70:67:\overline{10}|}$

   (d) $\ddot{a}^{(12)}_{70:67:\overline{10}|}$

   (e) $\ddot{a}_{\overline{70:67}}$

   (f) $\ddot{a}^{(12)}_{\overline{70:67}}$

7. State in words the meanings of the symbols A_{xy}, $A_{\overline{xy}:\overline{n}|}$ and $\bar{A}_{xy:\overline{n}|}$. Prove that:

   (a) $A_{\overline{xy}:\overline{n}|} = 1 - d\ddot{a}_{\overline{xy}:\overline{n}|}$

   (b) $\bar{A}_{xy:\overline{n}|} = 1 - \delta\bar{a}_{xy:\overline{n}|}$