9 Some Demographic Topics

9.1 Single Figure Indices

How do we compare the mortality experience of two or more populations?

A single figure that summarises the mortality experience of a population **enables easy comparison.**

Single Figure indices are formulas for **calculating such a single figure:**

Notes on single figure indices:

- they are easy to **assimilate, rather than a range of age-specific rates; but**
- there is a loss of information in **summarising and so care needs to be taken in the construction and application of the indices**

Crude Death Rate (CDR)

$$\text{CDR} = \frac{\text{Total deaths}}{\text{Total exposure}} = \frac{\sum_x d_x}{\sum_x \mathsf{E}_x^c}$$

Where:

$$d_x = \text{number of deaths in age group } x$$

$$\mathsf{E}_x^c = \text{“population in age group } x\text{”}$$

In the handout example we have:

Region A: $\quad \text{CDR} = \frac{175}{7,900} = 0.022$

Region B: $\quad \text{CDR} = \frac{267}{7,900} = 0.034$

Country: $\quad \text{CDR} = \dfrac{5,910}{357,000} = 0.017$

Notes:

(a)

$$
\begin{aligned}
\text{CDR} &= \frac{\sum_x d_x}{\sum_x \mathsf{E}^c_x} = \frac{\sum_x \mathsf{E}^c_x \frac{d_x}{\mathsf{E}^c_x}}{\text{Total population}} \\
&= \sum_x \left(\frac{\mathsf{E}^c_x}{\text{Total population}} \right) \mu_x \\
&= \sum_x w_x \, \mu_x
\end{aligned}
$$

Hence, the CDR is **a weighted average of the age-specific forces of mortality, where the weights reflect the population structure.**

(b) Strengths of the CDR:

- CDR is easy to **calculate**

- the data requirements **are minimal (just the total deaths and the total population).**

(c) Weaknesses of the CDR:

- CDR is heavily influenced by **the age structure of the population.**

In the handout example the CDRs imply that:

- Region B has **heavier mortality than Region A**
- Both regions have **heavier mortality than the country**

However, the age specific mortality indicate that:

$$\begin{aligned} & \text{Mortality Region A} \\ \approx\; & \text{Mortality Region B} \\ \approx\; & \text{Mortality of Country} \end{aligned}$$

CDR for Region A is influenced by the **relatively high proportion of young people.**

The country has, relatively, **an even younger population than either Region A or B.**

We need to develop an index which is less influenced by differences in age structures.

Standardised Death Rate (SDR)

$$\text{SDR} \quad = \quad \sum_x w_x \, \mu_x \qquad \text{(never SMR!)}$$

Where: $$w_x = \frac{{}^s\text{E}_x^c}{\sum_x {}^s\text{E}_x^c}$$

are based on a standard population

We note that:

(a) For Region A, (with age specific ${}^{A}\mu_x$):

$$\text{SDR}_A = \frac{\sum_x {}^{s}\mathsf{E}_x^c \; {}^{A}\mu_x}{\sum_x {}^{s}\mathsf{E}_x^c}$$

In the handout example (taking the country's population structure as standard), we have for SDR_A:

$$\frac{\frac{20}{2{,}000}55,000 + \frac{5}{1{,}500}50,000 + \cdots + \frac{50}{100}2,000}{55,000 + 50,000 + \cdots + 2,000}$$

$$= 0.0171$$

$\text{SDR}_B = 0.01689$

(b) The SDR is *not* very influenced by **the population structure as shown by:**

$$\textbf{SDR}_A \approx \textbf{SDR}_B$$

(c) The data requirements are:

- age specific **mortality rates of the region**
- age specific **structure of the standard population (weights)**

However, the weights (age specific structure of the standard population) may be **unknown or unreliably estimated.**

We consider the case where the weights are **only reliably known at specific times (eg census times).**

Indirectly Standardised Death Rate

The calculation of the SDR is an example of **direct standardisation.**

However the age specific structure of a standard population may only be known at **census dates.**

This gives problems if we need to calculate the SDR **at dates in between these censuses.**

Assumption:

$$\frac{\textbf{SDR (at census)}}{\textbf{CDR (at census)}} = \frac{\textbf{SDR (non-census)}}{\textbf{CDR (non-census)}}$$

Then we have that:

$$\begin{aligned}\text{SDR (non-census)} &= \frac{\text{SDR (at census)}}{\text{CDR (at census)}} \\ &\quad \times \text{CDR (non-census)} \\ &= F \times \text{CDR (non-census)}\end{aligned}$$

and the data requirements for CDR (non-census) are **easily met even at non-census times.**

F is called the **area comparability factor, and:**

$$F = \frac{\dfrac{\sum_x {}^s\mathsf{E}_x^c\,\mu_x}{\sum_x {}^s\mathsf{E}_x^c}}{\dfrac{\sum_x d_x}{\sum_x \mathsf{E}_x^c}}$$

F is calculated at **the census and assumed to remain constant until the next census when it is updated.**

Standardised Mortality Ratio

The SMR is a ratio (not rate) of:

$$\frac{\text{actual deaths in region}}{\text{expected deaths in region}}$$

The expected deaths are calculated using **the age specific mortality rates for the standard population.**

Therefore, the SMR for region A relative to a standard population S, is:

$$\text{SMR}_A = \frac{\sum_x {}^A\mu_x \, {}^A\mathsf{E}_x^c}{\sum_x {}^s\mu_x \, {}^A\mathsf{E}_x^c}$$

Notes:

(a) SMR is the **most common ratio found in demographic studies.**

(b) In our example, SMR_A: $$\frac{20+5+\cdots+50}{2,000\frac{530}{55{,}000}+1,500\frac{160}{50{,}000}+\cdots+100\frac{1{,}000}{2{,}000}}$$

$= 1.022 (= 102.2\%)$

$\text{SMR}_B = 1.009 (= 100.9\%)$

(c) The data requirements are:

- total number of **deaths in the region**
- population at **each age group in the region**
- age-specific **mortality rates in standard population**

This information is more likely to be **known more reliably than the information for the SDR (in particular the age specific population structure).**

(d) The SMR is usually expressed as a percentage and:

- a region which has the same mortality as **the standard population will have an SMR of 100%**
- if the SMR is greater than 100% then **the region has heavier mortality than the standard population.**
- if the SMR is less than 100% then **the region has lighter mortality than the standard population.**

Data for examples on single figure indices

Table 1: Mortality statistics

Age Group	Region A population	Region A deaths	Region B population	Region B deaths	Country population	Country deaths
0–9	2000	20	1000	10	55000	530
10–19	1500	5	1000	3	50000	160
20-29	1000	2	1000	2	60000	120
30–39	800	2	950	2	45000	95
40–49	700	2	900	2	45000	105
50–59	600	4	800	5	40000	250
60–69	500	10	800	16	30000	550
70–79	400	30	700	52	20000	1400
80–89	300	50	600	100	10000	1700
90+	100	50	150	75	2000	1000
Totals	7900	175	7900	267	357000	5910

Table 2: Age-specific mortality rates

Age group	Region A	Region B	Country
0–9	0.010	0.010	0.010
10–19	0.003	0.003	0.003
20–29	0.002	0.002	0.002
30–39	0.003	0.002	0.002
40–49	0.003	0.002	0.002
50–59	0.007	0.006	0.006
60–69	0.020	0.020	0.018
70–79	0.075	0.074	0.070
80–89	0.167	0.167	0.170
90+	0.500	0.500	0.500

9.2 Models for Population Projections

Projections of the population are needed to assist in the planning for:

- **demand for food, power, services, transport**
- **housing**
- **welfare services**
- **taxes**
- **labour costs**
- **national insurance contributions**

We consider mathematical models and component models for population projections.

Mathematical Models

Mathematical models:

- are projections based on **the total populations**
- take no account of

- **fertility rates**
- **mortality rates**
- **migration rates**

The advantages of mathematical models:

- data requirements **are minimal**
- explicit assumptions **can be kept to a minimum**
- **makes comparisons easy**

<u>The Exponential Model</u>

Definition:

$$P_t = \text{Size of population at time t} \quad (t \geq 0)$$

Then for the exponential we have:

$$P_t = P_0 \, \mathrm{e}^{r\,t}$$

For some growth parameter r.

Notes:

(a) The rate of grow is **proportional to its current size:**

$$\frac{d}{dt}P_t = \frac{d}{dt}P_0\,\mathrm{e}^{r\,t} = r\,(P_0)\,\mathrm{e}^{r\,t} = r \cdot P_t$$

(b) This is a very **simple model with only 1 parameter**

(c) The model can be **realistic, particularly over a short time period.**

(d) It takes account of **only the most basic information about a population — its size.**

(e) Since $P_t = P_0\,\mathrm{e}^{r\,t}$ then as $t \to \infty$:

if $r > 0$, then $\quad P_t \to \infty$

(population will continue rising indefinately)

if $r < 0$, then $\quad P_t \to 0$

(population can become extinct after some time)

Neither of these is considered realistic.

The Logistic Model

Populations do not normally increase indefinitely due to **limitations of resources.**

We adjust the exponential model by introducing a factor that **slows down the population growth as the population increases.**

For the logistic model we have:

$$\frac{d}{dt}P_t = r \cdot P_t - k \cdot P_t^2$$

Notes:

(a) The differential equation is satisfied by:

$$P_t = \frac{1}{c_1 \cdot \mathrm{e}^{-r\,t} + c_2} = \frac{\mathrm{e}^{r\,t}}{c_1 + c_2 \cdot \mathrm{e}^{r\,t}}$$

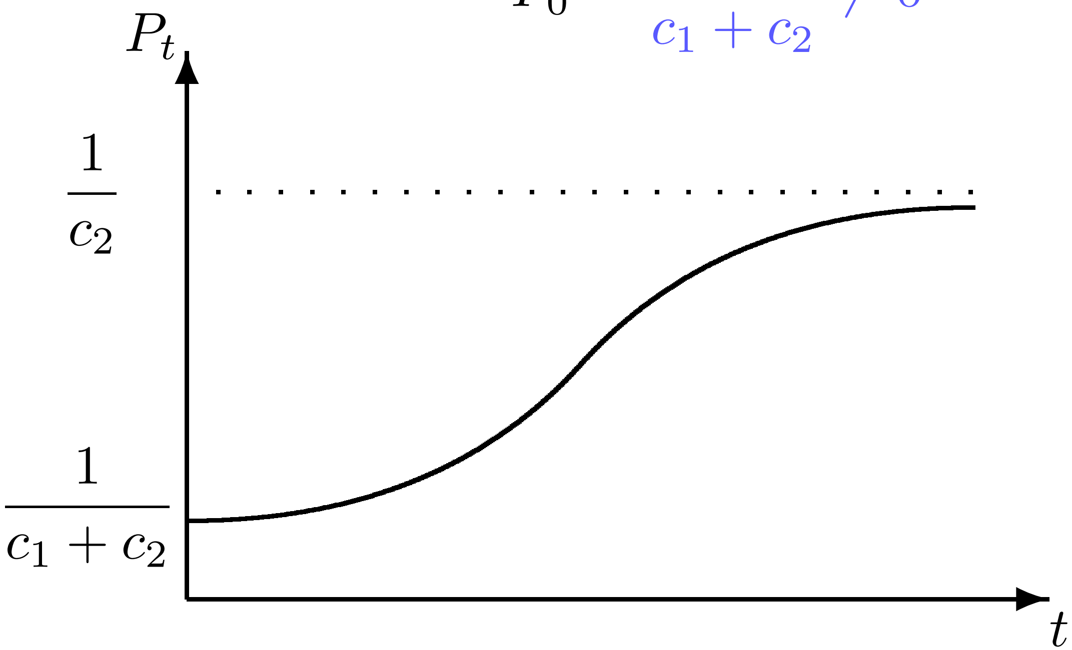
(b) The model has 3 parameters: r, c_1 and c_2. Therefore is should **give a better fit than the exponential model with only 1 parameter.**

(c) We have:

$$\lim_{t \to \infty} P_t = \frac{1}{c_2} < \infty$$

Also:

$$P_0 = \frac{1}{c_1 + c_2} \neq 0$$



(d) The parameters can be estimated from data by **least squares estimation or any other method.**

Problem with mathematical models

Projection of the components of the population separately (eg males, females) may **not give the same value as projecting the population as a whole.**

Component Projection Method

This method projects the population size *and* structure on the basis of:

- **the existing population**
- **future births**
- **future deaths**
- **future immigration/emigration**

The scheme would proceed as follows:

(i) Start with an estimated population **at time 0 subdivided by age, gender, etc.**

(ii) Project mortality, fertility, migration rates for **each age, gender etc. for each year into the future.**

(iii) Build up the population **year by year by applying mortality, fertility, migration rates to successively projected populations.**

In projecting the rates:

(a) Note that even though mortality rates are relatively stable, **future mortality rates are difficult to predict.**

Future deaths may depend on **future births, immigration/emigration.**

(b) Future births are difficult to predict. They depend on:

- number of **women of childbearing age**
- trends in **number of children (fashion, economics)**
- age of **mother at birth of first child**
- trends in **marriage/co-habiting**

(c) Future national immigration/emigration will depend on **future economic conditions and politics.**

Future local movements could **depend on employment prospects.**

— End —