7 Policy Design and Duration Dependence

7.1 Introduction

In this section, we look in more detail at features of disability insurance and long-term care insurance. We will find that this introduces duration dependence of two forms:

(1) Some of the transition intensities may depend on the **length of time (duration) spent in a state.** In the case **the Markov assumption does not hold.**

(2) Some of the **policy cashflows** may depend on the duration spent in a state. This can happen even if the underlying life-history model is Markov.

7.2 Features of disability insurance

Disability insurance is commonly called Income Protection Insurance (IPI) in the UK. It used to be known as Permanent Health Insurance (PHI) but this is obsolete. It is meant to pay a benefit in the form of **income during periods of incapacity due to sickness and disability.**

The history of IPI goes back to friendly societies (FSs) in the late 19th and early 20th centuries. This preceded the welfare state period and FSs provided small scale sickness and unemployment benefits, life assurance, funeral costs, etc., often in local communities. FSs still exist as small scale insurance companies.

In the mid 20th century the welfare state began to provide modest benefits in the event of sickness or permanent disability, so FSs were no longer so necessary as a ‘safety net’. However, there is still a need for sickness benefits for better paid workers. IPI is currently issued by many mainstream insurers.

In the last section we just assumed that premiums were payable continuously while able, benefits were payable continuously while sick, and death benefits might be paid as well. In practice, IPI policies are more complicated.

Term: The policies are typically **written up to the normal retirement age (NRA).**

Definition of sickness: The policy will specify the definition of sickness. A typical definition is:

"totally unable, through sickness or accident, to follow own occupation and not following any other for profit or reward."

Note that a definition like **"unable to follow any occupation" may be very restrictive to the policyholder.**

Exclusions: There may be exclusions to inability to work due to **pregnancy, AIDS, drug related illnesses etc.**

Premiums: The premiums are typically **level monthly and payable while benefit is not being received.** Premium rates are usually guaranteed at outset.

Benefits: Benefits are usually in the form of **a monthly (or weekly) income.** This may be fixed, or increase with inflation, or it may fall to a lower level (e.g. half) after some fixed duration of payment, to **encourage the policyholder to return to work.**

Waiver of Premiums: Since premiums are not paid while the benefit is received, this is also a benefit of **waiver of premiums.**

Deferred Period: Benefits do not, in fact, start as soon as the policyholder falls sick. They are deferred for a period of time, chosen by the policyholder, during which the policyholder must remain sick, or benefits will not commence. This is called **the deferred period.**We denote it d (in years). Typical values are **1 week, 4 weeks, 13 weeks, 26 weeks or 52 weeks, i.e.** $d \approx 1/52, 1/12, 1/4, 1/2$ **or 1 year.**

Waiting period: A period of time after taking out a IPI policy, during which **the new policyholder is not entitled to sickness benefits.**This may be 6 months. It is not the same as the deferred period.

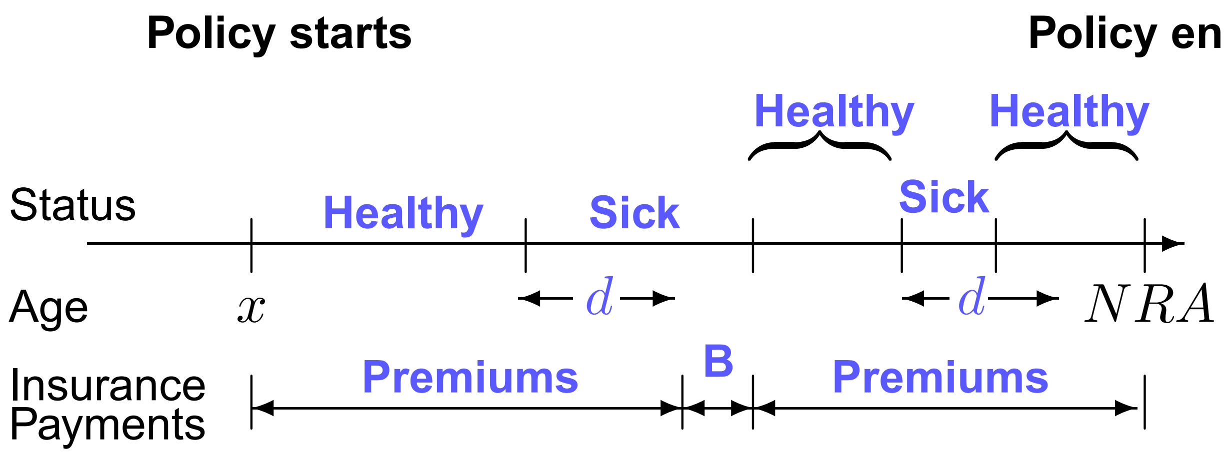
Off-Period: If the benefits are cut after some duration of continuous sickness (see above), the off-period defines the **minimum period of good health that must pass before two**
episodes of sickness can be considered as different.

Benefit Limits: The benefit will be reduced if the policyholder's total net income is greater than, say, 75% of net income before sickness. **This removes the chance that a**

policyholder will be better off sick than at work.

Underwriting: Medical underwriting is similar to, but not the same as, life insurance. In particular, more details about **occupation are considered for IPI underwriting than for life insurance underwriting.**

The figure below gives an overview of how IPI policies work.



Other features of IPI include:

- The claims experience **can be very volatile**.

- There is greater scope for moral hazard than in life insurance and hence **the need for greater claims control**.
- Policies are expensive and the need for IPI may not be clear and so **they may not be easy to sell**.

7.3 Duration dependence caused by the deferred period

Occupancy probabilities

By definition, the deferred period means that the payment of sickness benefit depends on the duration of sickness. This is true **even if the underlying life history model is Markov.** Therefore we define:

$$
\begin{aligned}
{}_{d,t}p_x^{12} \quad &= \quad \textbf{P}\big[\ \textbf{life is sick at age } x+t \textbf{ with duration} \\
&\qquad \textbf{of sickness} \leq d \,\big|\, \textbf{life is healthy at age } x\ \big]
\end{aligned}
$$

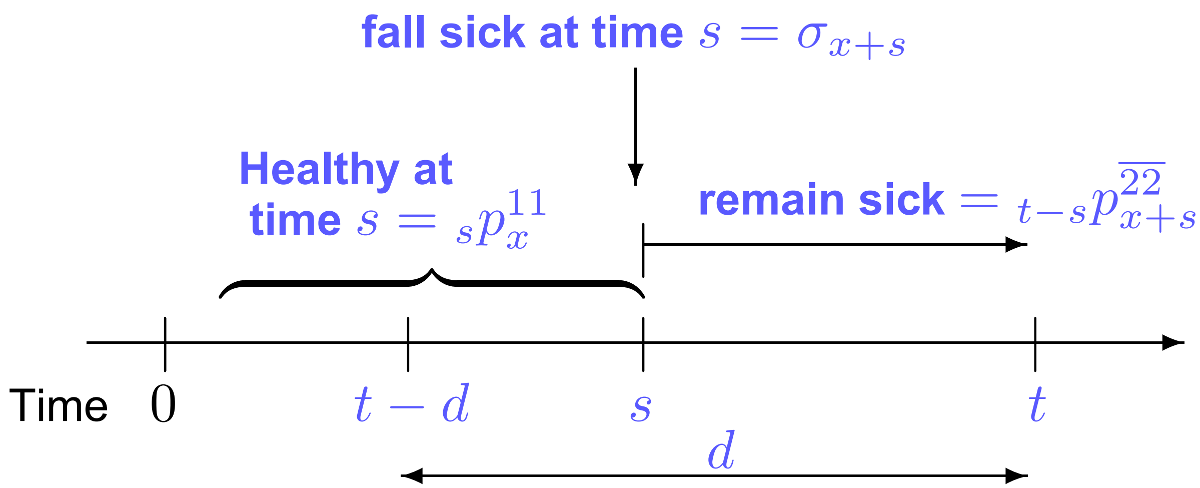
Since only the policy cashflows while sick are affected, we do not need to define similar probabilities for any other states.

To find an expression for this in terms of more basic quantities, we consider the ***last* time at which the policyholder fell sick.** This could have been any time between time $t-d$ and t. There are two cases:

Case 1: $d \geq t$. Since we knew anyway that the life last fell sick between time 0 and time t, knowing that the duration of sickness is $\leq d$ gives us no additional information, so:

$$_{d,t}p_x^{12} = {}_tp_x^{12}.$$

Case 2: $d < t$. The event (x) **fell sick for the *last* time at time** $s \leq d$requires (x) to be healthy at age $x+s$, to fall sick before age $x+s+ds$, and then to remain sick for suration $t-s$. See the following diagram:



This event has probability:

$$t - d \leq s \leq t$$

$$ {}_sp_x^{11}\, \sigma_{x+s}\, ds\, {}_{t-s}p_{x+s}^{\overline{22}} $$

and since s lies between $t-d$ and t:

$$ {}_{d,t}p_x^{12} = \int_{s=t-d}^{t} {}_sp_x^{11}\, \sigma_{x+s}\, {}_{t-s}p_{x+s}^{\overline{22}}\, ds. $$

This can be evaluated by **numerical integration.**

EPVs

When benefits are duration-dependent, Thiele's equations do not hold. However, it is still possible to calculate premiums and reserves in various ways, although we lose the extreme generality and flexibility that Thiele's equations give us.

We show here one way to calculate IPI premiums **when the policy has a deferred period.** It is by analogy with the EPV of a life annuity, payable continuously, which is:

$$EPV = \bar{a}_x = \int_0^\infty v^t \, {}_tp_x \, dt.$$

This has the following interpretation: the annuity dt is payable at time t if the **status** of being alive is fulfilled. This has probability ${}_tp_x$ and payment at time t is discounted by v^t.

Now apply the same reasoning to the following IPI policy issued to a healthy life aged x.

- The policy term is n years.
- Premiums at rate $\bar{P}$ per annum are payable continuously while the life is healthy **or** the life is sick with a duration less than or equal to d years.
- sickness benefit is payable continuously at rate $\bar{B}$ per annum while the life is sick with duration greater than d years.

Premiums are payable at time t as long as the status **'alive and not sick for longer than d years'** is fulfilled, which has probability $\left({}_tp_x^{11} + {}_{d,t}p_x^{12}\right)$.

Benefits are payable at time t as long as the status **'alive and sick for longer than d years'** is fulfilled, which has probability $\left({}_tp_x^{12} - {}_{d,t}p_x^{12}\right)$.

Summing (integrating) the discounted cashflows over the policy term we have:

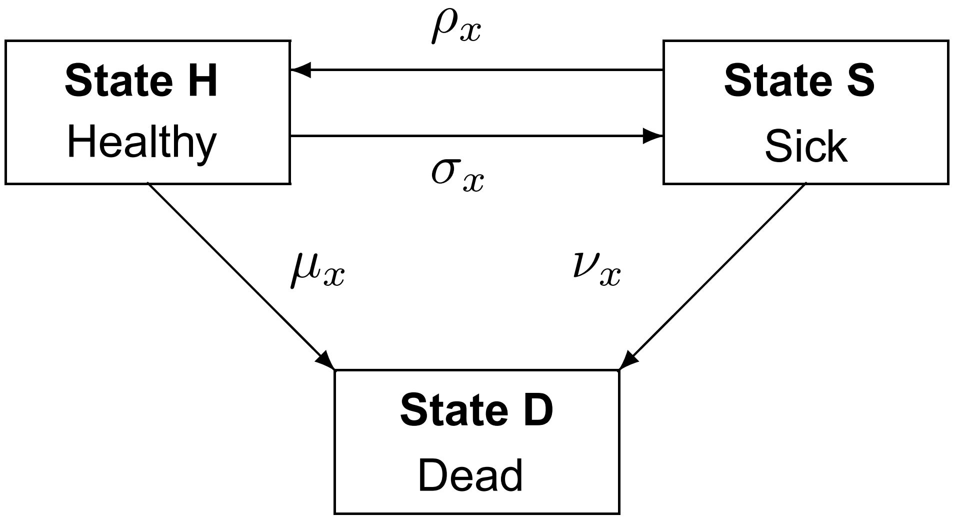
$$\bar{P} \int_{t=0}^{n} v^t \left({}_tp_x^{11} + {}_{d,t}p_x^{12} \right) dt =$$

$$\bar{B} \int_{t=0}^{n} v^t \left({}_tp_x^{12} - {}_{d,t}p_x^{12} \right) dt$$

from which $\bar{P}$ can be found. A similar approach can be used in many cases.

Example: Faculty and Institute of Actuaries: Subject 105 April 2002: Question 10.

The following 3 state model is used to price various sickness policies. The forces of transition σ, ρ, μ and ν depend only on age.



The following probabilities are defined:

${}_tp_x^{ij}$ is the probability that a life aged x in state i will be in state j at age $x+t$;

${}_tp_x^{\overline{ii}}$ is the probability that a life age x in state i will remain in state i until age $x+t$;

${}_tp_{x,z}^{ij}$ is the probability that a life aged x in state i will be in state j at age $x+t$, having been in state j for a period z

Using these probabilities and/or forces of transition, write down an expression for the expected present value of each of the following sickness benefits for a life currently aged

35 and healthy. The constant force of interest is δ.

(a) $1,000 per annum payable continuously while sick, but all benefits cease at age 65

(b) $1,000 per annum payable continuously while in the sick state for any continuous period in excess of a year. However, any benefit period is limited to 5 years payments, but the number of possible benefit periods is unlimited

(c) $1,000 per annum payable continuously throughout the first period of sickness only

Solution:

(a)

$$\text{EPV} = 1,000 \int_0^{30} \mathrm{e}^{-\delta\, t}\, {}_tp_{35}^{HS}\, dt$$

(b)

$$\text{EPV} = 1,000 \int_0^{\infty} \int_1^{6} \mathrm{e}^{-\delta\, t}\, {}_tp_{35,z}^{HS}\, dz\, dt$$

$$\text{or} \qquad 1,000 \int_0^{\infty} \mathrm{e}^{-\delta\, t}\, {}_tp_{35}^{HH}\, \sigma_{35+t} \int_1^{6} \mathrm{e}^{-\delta\, r}\, {}_rp_{35+t}^{\overline{SS}}\, dr\, dt$$

(c)

$$\text{EPV} \quad = 1,000 \int_0^{\infty} \mathrm{e}^{-\delta\, t}\, {}_tp_{35}^{\overline{HH}}\, \sigma_{35+t} \int_0^{\infty} \mathrm{e}^{-\delta\, r}\, {}_rp_{35+t}^{\overline{SS}}\, dr\, dt$$

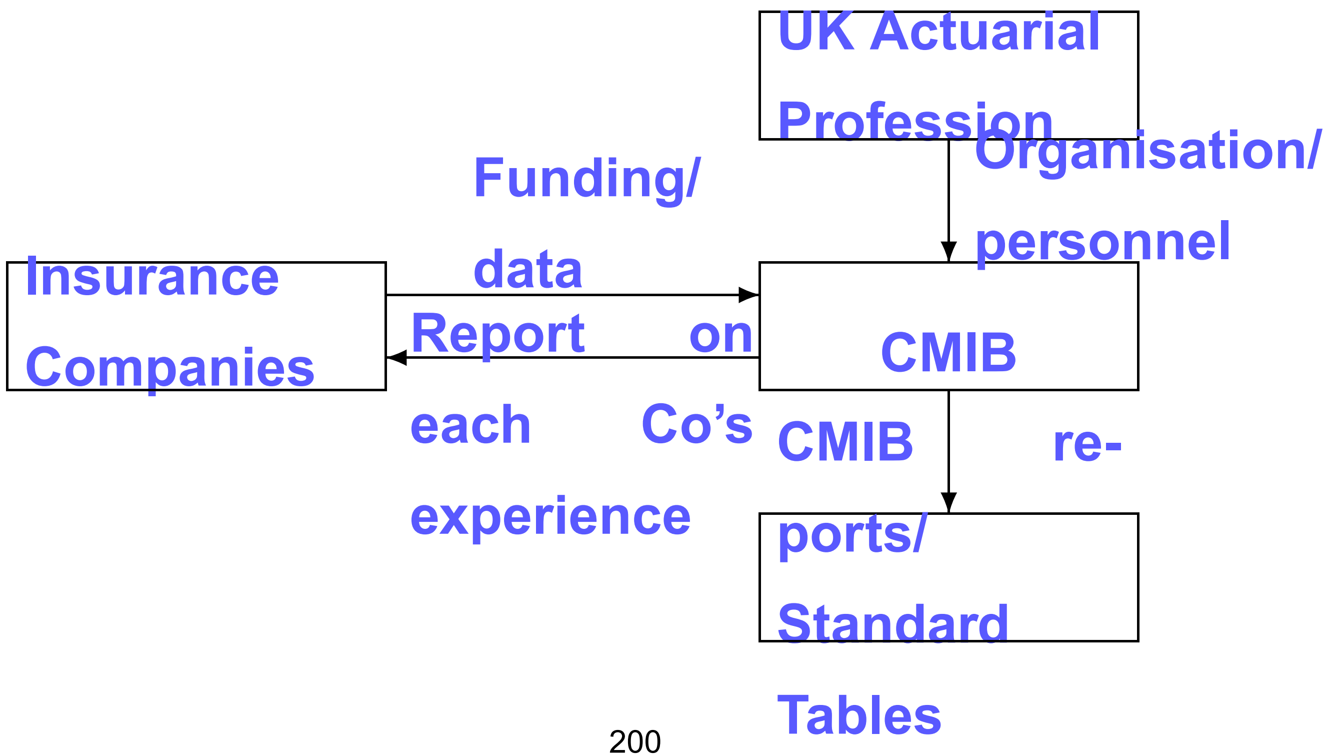
$$\text{or} \qquad 1,000 \int_0^{\infty} \mathrm{e}^{-\delta\, t}\, {}_tp_{35}^{\overline{HH}}\, \sigma_{35+t} \times \int_0^{\infty} \bar{a}_{\overline{r}|}\, {}_rp_{35+t}^{\overline{SS}} (\rho_{35+t+r} + \nu_{35+t+r})\, dr\, dt$$

7.4 Evidence for duration-dependent IPI intensities

There is strong evidence that the intensities of transitions **out of the ill state**depend on **how long a life has been ill**as well as on age.

A multiple-state model in which any intensity depends on the duration of stay in a state is called a **semi-Markov model**.

The strong evidence for duration-dependence comes from data collected by the Continuous Mortality Investigation Bureau (CMIB), which is a research bureau set up by the UK actuarial profession. Its structure is summarised in the diagram below:



The CMIB collects and analyses data from many UK life offices for the following classes of business:

- Life insurance
- Critical illness insurance
- Income protection insurance.

For IPI, the CMIB receives about **2/3 of all UK data.** To indicate the volume of data involved, Table 2 shows the numbers of **claim inceptions** and **recoveries** reported in 1987–94, males and females combined. (There are fewer recoveries than claim inceptions because: (a) **sickness can be terminated by other reasons like death;** and (b) **during a period on expanding new business new claims will exceed recoveries.**

Table 2: Numbers of events in CMIB IPI data 1987–94.

Deferred Period (weeks)	Claim Inceptions	Claim Terminations (Recoveries)
D1	30,311	12,409
D4	5,707	3,660
D13	3,195	1,794
D26	2,516	676
D52	811	139
Total	42,540	18,678

We now focus on data for males from 1975–87, which have been reported at great length (Source: CMI Report No.12 (1991).

Mortality from healthy, μ_x:

The CMIB did not collect data on deaths of healthy policyholders, so assumed that a suitable life table applied (based on the Males, Permanent Assurances, 1979–82 investigation). Premiums and reserves **do not depend strongly on** μ_x **anyway.**

Table 3: Sample mortality values from different tables.

Mortality	Sickness Duration	Age 30	Age 50
μ_x CMIR No 12	n/a	0.00042	0.00235
$\mu_{[x]}$ A1967–70	n/a	0.00037	0.00235
μ_x A1967–70	n/a	0.00065	0.00425
CMIR No 12	0	0.0415	0.0593
CMIR No 12	15 weeks	0.1108	0.1507
CMIR No 12	1 year	0.0627	0.0874
CMIR No 12	5 years	0.0190	0.0303

Mortality from sick, ν_x:

There was very little data. For comparison we consider some sample intensities in Table 3 and note the following:

- Mortality from sick increases with duration (up to about 15 weeks), then decreases and finally increases (not shown).
- Mortality from sick is **much greater than mortality from able, as expected.**
- Mortality does not depend strongly on deferred period (not shown).

Onset of sickness, σ_x:

Rates of 'onset' of sickness **depended on the deferred periods,**see Table 4 for examples.

Table 4: Sample values for ‘sickness’ transition intensities.)

Age	Sickness Intensities σ_x	
x	D1	D26
30	0.326	0.113
45	0.266	0.100
60	0.300	0.131

We note that:

- For both D1 and D26, the sickness intensities σ_x **do not change much with age.** This is slightly odd: **is it realistic that a 60-year old is equally likely to fall sick in a short time interval as a 30-year old?**

- The sickness intensities at D1 are roughly **3 times the intensities at D26.**

Claim inceptions:

We note that while the model is specified in terms of sickness intensities, the CMIB is only able to observe **claim inceptions,**which are not the same thing because of the deferred period.

Consider a healthy life age x with an IPI policy with deferred period d years. Any episode of episode of duration less than d years **will not lead to a claim inception.**

Given that to make a claim a life must first fall sick and then remain sick for duration d, claim inception intensities at age $x + t + d$ should be given by:

$$\sigma_{x+t} \, {}_{d}p_{x+t}^{\overline{22}}.$$

Sample values are given in Table 5.

Table 5: Sample values of ‘claim inception’ intensities.

	Deferred Period	
Age	D1	D26
30	0.126	0.00041
45	0.127	0.00146
60	0.173	0.00793

Unlike sickness intensities, these ‘claim inception’ intensities **increase with age (sharply for older ages).**

Recovery from sick, ρ_x:

Sample values of the recovery intensities (same for all deferred periods) are shown in Table 6.

Table 6: Sample values of ‘recovery from sick’ transition intensities.

	Age at Onset	
Duration	30	50
1 week	45.67	25.86
4 weeks	16.91	13.10
13 weeks	6.70	4.39
26 weeks	2.77	1.59
1 year	0.77	0.37
2 years	0.37	0.16

We note the following:

- Recovery rates change **more quickly with duration, z, than with age x.** This adds to the strong evidence for duration-dependence.

- The rapid change of recovery rates with duration will force us to use **a small step size for numerical calculations.** The CMIB used $1/3$ week.

Comparison over time:

In Table 7 we compare the ratio:

$$\frac{\textit{Actual}\ \textbf{number of claims (or recoveries)}}{\textit{Expected}\ \textbf{number of claims (or recoveries)}}$$

over different periods, where the expected numbers are based on the Males, Individual Policies, 1975–78 experience.

Table 7: Comparison of 'Actual/Expected' ratio over time.

	Males				Females			
	Claims		Recoveries		Claims		Recoveries	
Period	D1	D26	D1	D26	D1	D26	D1	D26
1987/90	109	137	95	56	142	350	92	51
1998	88	124	98	44	128	289	92	46

We note that:

- Male and female recovery experience is similar but **claims for females are much higher.**
- Claim inceptions are **volatile.**

- D1 recoveries are **reasonably stable.**

- D26 recoveries are **much worse than for 1975/78.**

Comparison between companies:

We compare the 5 largest contributors of data to the CMIB in 1987–94, using deferred period 4 weeks, see Table 8.

Table 8: Comparison of 'Actual/Expected' ratio between companies.

Company	Claims %	Recoveries %
A	82	73
B	64	58
C	91	72
D	102	61
E	58	46

We see large differences, for example:

Company D has **high numbers of claims and low recovery rates**

Company E has **low numbers of claims but low recovery rates**

These differences may be due to differences in between the companies in terms of:

- **underwriting procedures**
- **target market**
- **claims control.**

7.5 The CMIB semi-Markov model for IPI

All the evidence of the last section points to transition intensities out of the sick state that depens on duration as well as age. That is, the model in Figure 9, where:

x	represents **age at policy inception**
$x+t$	represents **current age**
z	represents **current duration of sickness.**

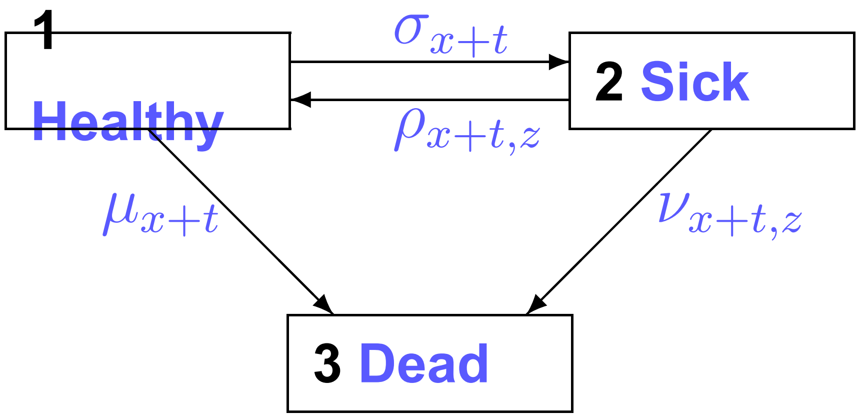


Figure 9: The CMIB's semi-Markov IPI model.

$\rho_{x+t,z}$ is interpreted as:

$$\rho_{x+t,z}\,dt \quad \approx \quad \textbf{P}\big[\ \textbf{life age } x+t\textbf{, sick for}$$

duration z will recover before

age $x + t + dt$]

The data suggest we fit separate models for:

- **males and females**
- **different deferred periods**
- **different occupational groups.**

To implement the model, we require:

(a) Parameterisation of the model from appropriate data. This involves **estimating the transition intensities.**

(b) Formulae for probabilities in terms of **the transition intensities.**

(c) To evaluate the probabilities **— numerical**

algorithms.

(d) Convenient (tabulated/ computer package) functions to **calculate premiums and reserves.**

Determining occupancy probabilities

We define some new notation:

$$
\begin{aligned}
{}_tp_{x,z}^{gh} &= \text{P}\big[\text{ life is in state } h \text{ at age } x+t \mid \text{life is in state } g \text{ at age } x \text{ with duration } z\big] \\
{}_{d,t}p_{x,z}^{gh} &= \text{P}\big[\text{ life is in state } h \text{ at age } x+t \text{ with duration } \leq d \mid \text{life is in state } g
\end{aligned}
$$

$$
\begin{aligned}
& \text{at age } x \text{ with duration } z \,] \\
{}_tp_{x,z}^{\overline{gg}} \quad = \quad & \mathrm{P}[\text{ life stays in state } g \text{ from } x \to x+t \mid \\
& \text{life is in state } g \\
& \text{at age } x \text{ with duration } z \,].
\end{aligned}
$$

Notes:

- ${}_tp_{x,z}^{gh} = {}_{\infty,t}p_{x,z}^{gh} = {}_{t+z,t}p_{x,z}^{gh}$
- if $g = 1$, then the duration z **is irrelevant and we write:**

$$ {}_{d,t}p_x^{12}, \quad {}_tp_x^{13}, \quad {}_tp_x^{11}, \quad {}_tp_x^{\overline{11}} $$

- if $g = 2$ and $z = 0$ we write:

$$ {}_tp_x^{21}, \quad {}_{d,t}p_x^{22}, \quad {}_tp_x^{\overline{22}}. $$

Deriving ${}_tp_x^{\overline{11}}$ and ${}_tp_{x,z}^{\overline{22}}$

(a) The transition intensities out of the healthy state are identical to those for the Markov model. Hence we have the same results:

$$\frac{d}{dt}\,{}_tp_x^{\overline{11}} = -\,{}_tp_x^{\overline{11}}(\sigma_{x+t} + \mu_{x+t})$$

$${}_tp_x^{\overline{11}} = \exp\left(-\int_0^t (\sigma_{x+r} + \mu_{x+r})dr\right).$$

(b) Similarly we note that:

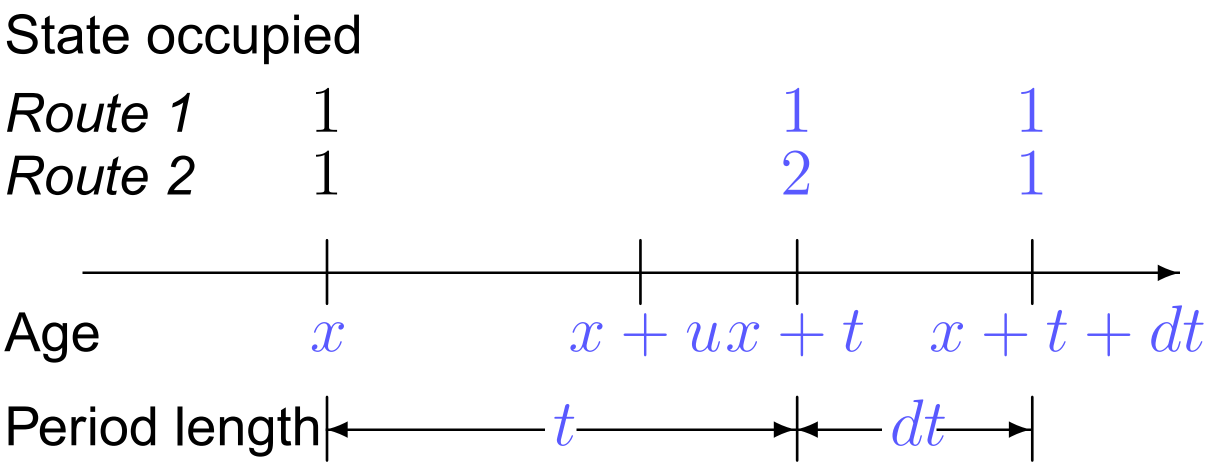
$$_{t+dt}p_{x,z}^{\overline{22}} = {}_{t}p_{x,z}^{\overline{22}} \times {}_{dt}p_{x+t,z+t}^{\overline{22}}$$

Which leads to an ODE with solution:

$$_{t}p_{x,z}^{\overline{22}} = \exp\left(-\int_{0}^{t}(\rho_{x+r,z+r} + \nu_{x+r,z+r})dr\right).$$

Deriving $_{t}p_{x}^{11}$

To derive an expression for $_{t}p_{x}^{11}$, consider the two possible routes illustrated in the figure below:



Where time u represents the **final time a life falls sick before age $x + t$**,in case they are sick at age $x + t$ (Route 2).

It is important to appreciate that dt is a short enough interval that we can assume **only one movement is possible**(up to terms with probability $o(dt)$ that we can ignore). Therefore:

- a life healthy at $x + t$ and at $x + t + dt$ must have **remained healthy throughout.**

- a life healthy at $x+t$ and sick at $x+t+dt$ must have **made exactly one movement.**

However, t is an extended period, unlike dt. Therefore:

- a life healthy at x and at $x+t$ could have **stayed healthy throughout or have had many episodes of sickness and recovery as long as they have recovered by $x+t$.**
- Similarly a life healthy at x and sick at $x+t$ could have **had one episode of sickness or many episodes of sickness, recovery and sickness again, as long as they are sick at $x+t$.**

Therefore we can write:

$$_{t+dt}p_x^{11} \quad = \quad \text{P}[\text{ life is healthy at } x+t\ ]$$

$$
\begin{aligned}
&\times \text{P}\big[\text{ life stays healthy in } x+t \\
&\qquad\qquad \longrightarrow \ x+t+dt\big] \\
&+ \text{P}\big[\text{life is sick at } x+t\ \big] \\
&\quad \times \text{P}\big[\text{life recovers in } x+t \\
&\qquad\qquad \longrightarrow \ x+t+dt\big]
\end{aligned}
$$

$$
\begin{aligned}
= \quad & {}_tp_x^{11} \left[1 - \left(\mu_{x+t} + \sigma_{x+t}\right) dt + \text{o}(dt)\right] \\
& + \int_0^t {}_up_x^{11}\, \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{\overline{22}}\, \rho_{x+t,t-u}\, du\, dt \\
& + \text{o}(dt).
\end{aligned}
$$

Rearranging:

$$\frac{{}_{t+dt}p_x^{11} - {}_tp_x^{11}}{dt}$$

$$\begin{aligned} &= -{}_tp_x^{11}\left(\mu_{x+t} + \sigma_{x+t}\right) \\ &\quad + \int_0^t {}_up_x^{11}\,\sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{\overline{22}}\,\rho_{x+t,t-u}du \\ &\quad + \frac{\mathsf{o}(dt)}{dt} \end{aligned}$$

and on letting $dt \longrightarrow 0$ we get the differential equation:

$$\frac{d}{dt}{}_tp_x^{11} = -{}_tp_x^{11}\left(\mu_{x+t} + \sigma_{x+t}\right)$$

$$+\int_0^t {}_u p_x^{11}\,\sigma_{x+u}\cdot {}_{t-u}p_{x+u}^{\overline{22}}\,\rho_{x+t,t-u}du.$$

Deriving ${}_{w,t}p_x^{12}$

Reminder:

$${}_{w,t}p_x^{12} \quad = \quad \text{P}\big[\text{ life is sick at age } x+t \text{ with duration } \le w \;\big|\; \text{healthy at age } x\,\big].$$

Now consider $\frac{d}{dw}{}_{w,t}p_x^{12}$ under two cases:

(i) If $w \geq t$, then:

$${}_{w,t}p_x^{12} = {}_t p_x^{12}$$

and so does not depend on w. Hence:

$$\frac{d}{dw}\left({}_{w,t}p_x^{12}\right) = 0 \quad \textbf{if} \quad w \geq t.$$

(ii) If $w < t$, then the duration of sickness is less than the time interval t. Therefore, the sickness has to start between ages:

$$x + t - w \quad \textbf{and} \quad x + t.$$

To derive the differential equation we introduce a small interval of time dwsuch that:

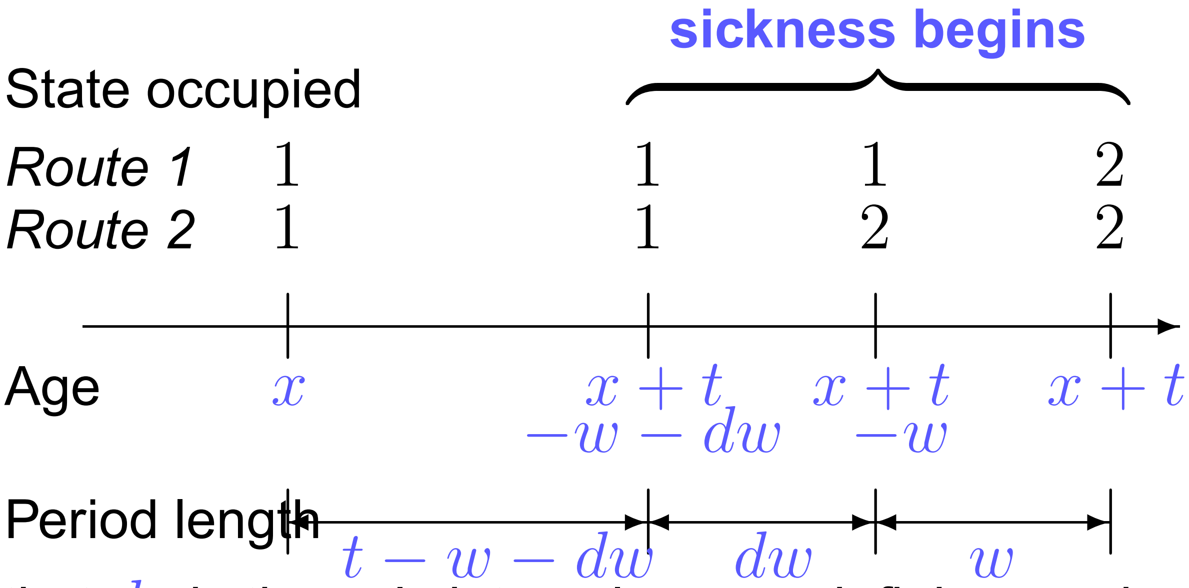
$$0 < dw < t - w.$$

So, we have:

$$0 \leq w < w + dw < t$$

and we consider the probability:

$$_{w+dw,t}p_x^{12}.$$



Remembering that dw is the only interval we are defining as short, we have:

$$_{w+dw,t}p_x^{12}$$

$$= \quad \mathbf{P}\Big[\text{ sick at age } x+t \text{ with duration } \le w + dw \,\Big|\, \text{healthy at age } x\Big]$$

$$= \quad \mathbf{P}\Big[\text{ sick at age } x+t \text{ with duration } \le w \,\Big|\, \text{healthy at age } x\Big]$$

$$+\mathbf{P}\Big[\text{ sick at age } x+t \text{ with duration between } w \text{ and } w+dw \,\Big|\, \text{healthy at age } x\Big]$$

$$= \quad {}_{w,t}p_x^{12}$$

$$+ \Big\{ {}_{t-w-dw}p_x^{11} \left[\sigma_{x+t-w-dw}\, dw + \mathbf{o}(dw)\right]$$

$$\times \left({}_{w}p_{x+t-w}^{\overline{22}} + o(dw)\right)\Big\}.$$

Rearranging gives:

$$\frac{{}_{w+dw,t}p_x^{12} - {}_{w,t}p_x^{12}}{dw}$$

$$= {}_{t-w-dw}p_x^{11} \cdot \sigma_{x+t-w-dw} \cdot {}_{w}p_{x+t-w}^{\overline{22}} + \frac{o(dw)}{dw}$$

and taking limits as $dw \to 0$, we get:

$$\frac{d}{dw}{}_{w,t}p_x^{12} = {}_{t-w}p_x^{11} \cdot \sigma_{x+t-w} \cdot {}_{w}p_{x+t-w}^{\overline{22}}.$$

Therefore:

$$\frac{d}{dw}{}_{w,t}p_x^{12} = \begin{cases} {}_{t-w}p_x^{11}\,\sigma_{x+t-w} \cdot {}_{w}p_{x+t-w}^{\overline{22}} & 0 \leq w < t \\ 0 & w \geq t. \end{cases}$$

For more formulae see Tutorial.

Numerical evaluation of occupancy probabilities

Again, we consider simple approaches based on Euler schemes, assuming the transition intensities to be known functions of **age, x, and/or duration, z.**

Computing ${}_tp_x^{\overline{11}}$ and ${}_tp_x^{\overline{22}}$

Recall that:

$$
\begin{aligned}
{}_tp_x^{\overline{11}} &= \exp\left(-\int_0^t (\sigma_{x+r} + \mu_{x+r})dr\right) \\
{}_tp_{x,z}^{\overline{22}} &= \exp\left(-\int_0^t (\rho_{x+r,z+r} + \nu_{x+r,z+r})dr\right).
\end{aligned}
$$

We can use direct or numerical integration to solve these depending on the form of the transition intensities.

Computing ${}_tp_x^{11}$

Recall that:

$$
\frac{d}{dt}{}_tp_x^{11} = -{}_tp_x^{11}\left(\mu_{x+t} + \sigma_{x+t}\right)
$$

$$+ \int_0^t {}_u p_x^{11}\, \sigma_{x+u} \cdot {}_{t-u} p_{x+u}^{\overline{22}}\, \rho_{x+t,t-u} du.$$

We have the boundary condition:

$$ {}_0 p_x^{11} = 1 $$

and we choose a step size, h(the CMIB used stepsize $1/156$ **year).**

Using Euler's metheod, we approximate ${}_h p_x^{11}$ as:

$$ {}_h p_x^{11} \approx {}_0 p_x^{11} + h \left(\frac{d}{dt} {}_t p_x^{11} \bigg|_{t=0} \right). $$

By noting that:

$$\left.\frac{d}{dt}{}_tp_x^{11}\right|_{t=0} = -{}_0p_x^{11}\left(\mu_x+\sigma_x\right) + \int_0^0 {}_up_x^{11}\,\sigma_{x+u}\cdot{}_{t-u}p_{x+u}^{\overline{22}}\,\rho_{x,-u}du = -\left(\mu_x+\sigma_x\right)$$

we have:

$${}_hp_x^{11} = 1 - h\cdot\left(\mu_x+\sigma_x\right).$$

The next Euler step is:

$${}_{2h}p_x^{11} \approx {}_hp_x^{11} + h\left(\left.\frac{d}{dt}{}_tp_x^{11}\right|_{t=h}\right)$$

where:

$$\left.\frac{d}{dt}\,{}_{t}p_x^{11}\right|_{t=h} = -{}_{h}p_x^{11}\left(\mu_{x+h}+\sigma_{x+h}\right) + \int_0^h {}_{u}p_x^{11}\,\sigma_{x+u}\cdot{}_{h-u}p_{x+u}^{\overline{22}}\,\rho_{x+h,h-u}\,du$$

and so on.

<u>Calculating ${}_{w,t}p_x^{12}$</u>

For ${}_{w,t}p_x^{12}$, using Euler's method we obtain:

$${}_{s,s}p_x^{12} = {}_{0,s}p_x^{11}\,\sigma_{x+s}\,s$$

$$
\begin{aligned}
{}_{s,2s}p_x^{12} &= {}_{0,2s}p_x^{11}\,\sigma_{x+2s}\,s \\
{}_{2s,2s}p_x^{12} &= {}_{s,2s}p_x^{12} + {}_{s}p_x^{11}\,\sigma_{x+s}\,s\,{}_{s}p_{x+s}^{\overline{22}}
\end{aligned}
$$

and so on (see separate note explaining the logic).

Determining EPVs

The same approach as we used to deal with deferred periods can be extended to the semi-Markov model. Consider, for example, the following policy issued to a life aged x:

- premiums ceasing at age 65
- premiums at rate $\bar{P}$ per annum payable continuously while healthy **and**

waived while the benefit is payable

- benefits at rate $\bar{B}$ per annum payable **continuously while sick with duration $\geq d$**
- interest at i per annum effective
- no expenses.

The equation of value is:

$$P \int_{t=0}^{65-x} v^t \left({}_tp_x^{11} + {}_{d,t}p_x^{12} \right) dt =$$

$$\bar{B} \int_{t=0}^{65-x} v^t \left({}_{t,t}p_x^{12} - {}_{d,t}p_x^{12} \right) dt$$

We now suppose that the annual rate of sickness benefit is:

$$\begin{array}{rll} 0 & \text{for} & \text{duration } \leq d_1 \\ \bar{B}_1 & \text{for} & d_1 < \text{duration } \leq d_1 + d_2 \\ \bar{B}_2 & \text{for} & \text{duration } \geq d_1 + d_2 \end{array}$$

with premiums waived while any benefit is payable. The equation of value is then:

$$\bar{P} \int_{t=0}^{65-x} v^t \left({}_tp_x^{11} + {}_{d_1,t}p_x^{12} \right) dt$$

$$= \quad \bar{B}_1 \int_{t=0}^{65-x} v^t \left({}_{d_1+d_2,t}p_x^{12} - {}_{d_1,t}p_x^{12} \right) dt$$

$$+ \bar{B}_2 \int_{t=0}^{65-x} v^t \left({}_{t,t}p_x^{12} - {}_{d_1+d_2,t}p_x^{12} \right) dt.$$

7.6 Long-term care insurance

Background

Long-term care (LTC) insurance is insurance designed to **fund or partially fund long-term care.**

Long-term care is care provided to those people who are no longer able to look after themselves, typically the elderly or infirm.

LTC can:

- range from 2 hours a week to 24 hours a day
- be informal, from a spouse or children (usually in the home)
- be formal, on a paid-for basis, typically provided in a nursing home or residential home.

LTC is a *protection product* with the aim of protecting against **the costs of LTC.**The average cost of a room (2003 figures) in:

- a private nursing home is $23,690 p.a.
- a private residential home is $17,115 p.a..

In the UK, about 70% of LTC is paid for by the state, and about 30% privately. State LTC is means-tested so that, generally, if someone needing care can pay for it **then they should.**

It has been estimated that the demand for LTC will **increase substantially in the next 30 years, as the table below illustrates:**

Level of	No. lives (000s) in:			
Care	2001	2011	2021	2031
Low	2,392	2,602	2,844	3,041
Moderate	2,082	2,161	2,366	2,461
Regular	1,564	1,720	1,925	2,141
Continuous	706	840	993	1,185
Total	6,745	7,324	8,098	8,828

Nutall *et al.* (1993)

This is primarily due to:

(i) **improving mortality; and**

(ii) **the ageing population.**

More important may be the costs of future care, which have been projected to **increase from:**

- **$12 billion**in 1995 (paid for: 27% privately, 73% state); to
- **$33.5 billion**in 2031 (paid for: 61% privately, 39% state.)

LTC insurance started to appear in the UK in the early 1990s and take-up of policies has been very slow. Market size has been estimated at about 25,000 to 35,000 policies.

Product Design

There is no one single LTC Insurance product. We describe some of the more common designs here:

Stand-alone: Premiums can be **single lump-sum or regular until time of**

claim or death.

Benefits are in the form of **income for the duration of a valid claim**

LTC as a rider to a whole life plan: In the event of satisfying the claims criteria the death

benefit is **accelerated and payable in monthly installments**

LTC as an extension to IPI: Before NRA, IPI claims criteria and benefit level are used. After this age, LTC claims criteria used are with **the same or increased level of benefits.**Premiums may remain at the same level, decrease, or stop at the change-over age.

Immediate care annuities: These are effectively impaired life annuities with the aim of

covering the costs of current care or care that will be required in the near future.A single lump-sum premium provides a guaranteed monthly annuity to cover part or all of care costs **for as long as care is required.**

Product Features

Claims Criteria

Claims can usually be triggered through **physical disability or cognitive impairment.**

Typical claims criteria would be:

- **failing 2 or 3 out of a benchmark of 6 activities of daily living (ADLs); or**
- **failing a cognitive impairment test.**

The Association of British Insurer's (ABI's) benchmark ADLs are: washing, dressing, feeding, toileting, mobility and transferring.

LTC insurance products may pay:

- benefits towards care costs **up to a maximum sum assured; or**
- a fixed % of the sum assured on triggering of benefits. For example:

  50% of SA **on the failure of 2 ADLs**

  100% of SA **on the failure of 3 or more ADLs.**

Failure of 3 ADLs is typically used as the point for payment of the full sum assured, since this is the point when **residential care will usually be required.**

Benefit Limits

Limits may be imposed on:

- **the length of the benefit period (eg 3 or 5 years)**
- **the total amount of benefits payable.**

Benefit Escalation

Benefits may be level or indexed. Types of indexation:

- **fixed % per annum (eg 5%)**
- **linked to an index, usually with a cap (eg RPI up to a maximum of 15% p.a.).**

Premiums

Premiums can be **regular or single lump-sum.**

Regular premium policies are generally reviewable (ie insurance company can increase them at their discretion).

Guarantees are sometimes offered, for example, the premium rates may be guaranteed to **remain level for 10 years, but annually reviewable thereafter.**

It is normal for regular premiums to **escalate in line with the benefits.**

Premium Waiver

Waiver of premium applies to all regular premium contracts, such that premiums are **not payable while benefits are being paid.**

Deferred Period

This is the period of time a claimant must **continuously fail the claims criteria before benefits are paid.**

Typically this is 3 months, although there are also deferred periods of 6, 12, 24 and 36 months.

Other features

Since LTC is a protection product there is not normally any **surrender value or death benefit.**

Pricing long-term care products

We describe two approaches to valuing benefits in a LTC insurance contract:

(i) Multiple-state model approach

(ii) Inception/annuity approach.

Multiple-state model approach

We have already described this in detail. Here, we give another example to show that the approach can deal with the complexity of LTC contracts.

Consider the following LTC policy, with **premiums payable continuously at annual rate $\bar{P}$** and:

- SA of $B pa
- 50% of SA payable on loss of 2 ADLs
- 100% of SA payable on loss of 3 or more ADLs.

We represent the life history underlying this contract using the following model.

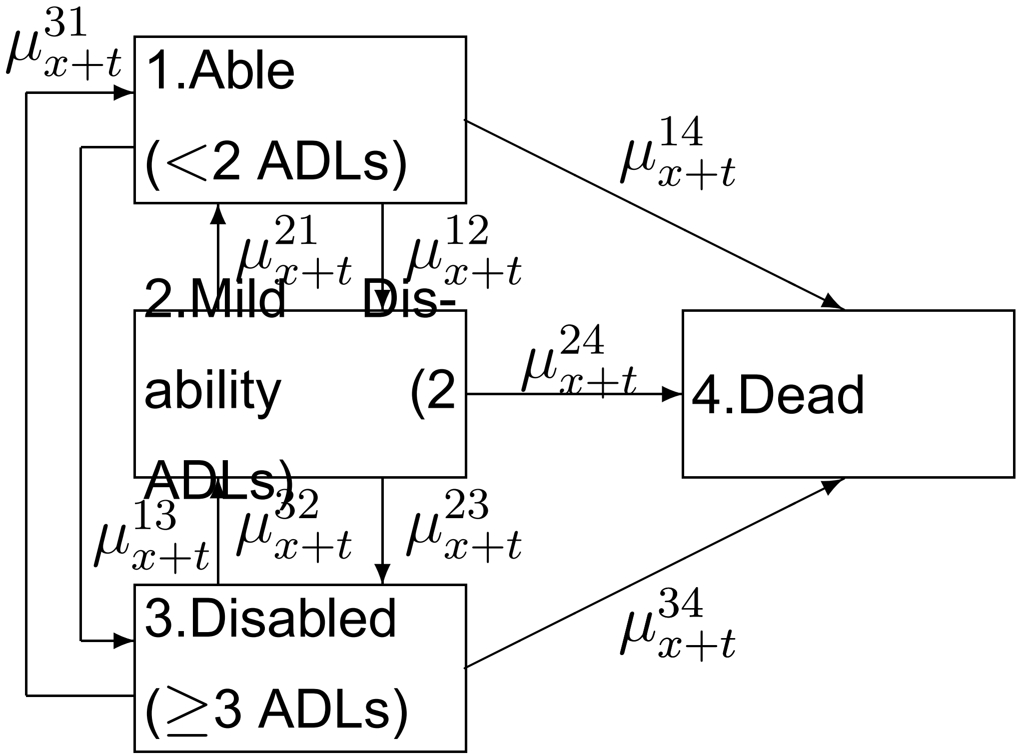


Figure 10: A Markov model of long-term care insurance, allowing for mild disability.

We can calculate the premium rate $\bar{P}$ from Thiele's differential equations:

$$
\begin{aligned}
\frac{d}{dt}V^1(t) &= \delta\, V^1(t) + \bar{P} - \mu_{x+t}^{12}\,(V^2(t) - V^1(t)) \\
&\quad -\mu_{x+t}^{13}\,(V^3(t) - V^1(t)) + \mu_{x+t}^{14}\,V^1(t) \\
\frac{d}{dt}V^2(t) &= \delta\, V^2(t) - \frac{B}{2} - \mu_{x+t}^{21}\,(V^1(t) - V^2(t)) \\
&\quad -\mu_{x+t}^{23}\,(V^3(t) - V^2(t)) + \mu_{x+t}^{24}\,V^2(t) \\
\frac{d}{dt}V^3(t) &= \delta\, V^3(t) - B - \mu_{x+t}^{31}\,(V^1(t) - V^3(t)) \\
&\quad -\mu_{x+t}^{32}\,(V^2(t) - V^3(t)) + \mu_{x+t}^{34}\,V^3(t) \\
\frac{d}{dt}V^4(t) &= 0
\end{aligned}
$$

using the boundary conditions:

$$V^i(\omega - x) = 0 \quad \textbf{for} \quad i = 1, 2, 3, 4$$

where ω is the oldest age in the life table.

Inception/annuity approach

This method assumes that once a life is claiming, they cannot **recover to a non-claiming state.**

We first introduce some notation:

- $i_x^{(z \text{ ADLs})}$ is the probability that **a life aged x fails $\geq z$ ADLs.**
- ${}_tp_x^{(<z \text{ ADLs})}$ is the probability that **a life aged x survives to time t without failing z or more ADLs.**
- ${}_tp_{x,d}^{(\geq z \text{ ADLs})}$ is the probability that **a life aged x with current duration d of having failed at least z ADLs survives to time t.**

Consider the following LTC contract issued to a life aged x:

- SA \$1 payable on failing z or more ADLs.
- Deferred period d years.

Then the EPV of the benefits, assuming they are just about to commence is:

$$a_{x,d}^{(z \text{ ADLs})} = \sum_{t=0}^{\infty} v^t \, {}_tp_{x,d}^{(\geq z \text{ ADLs})}$$

and the unconditional EPV of the benefits is:

$$\sum_{t=0}^{\infty} v^t \, {}_tp_x^{(<z \text{ ADLs})} i_{x+t}^{(z \text{ ADLs})} \, {}_dp_{x+t,0}^{(\geq z \text{ ADLs})} v^d a_{x+t,d}^{z \text{ ADLs}(d|all)}.$$

These can easily be calculated using **tabulated values and a spreadsheet.**

If the policy is more complex, then we may be able to value it as separate policies. For example, if:

- 50% of the benefit is payable on failure of 2 ADLs; and
- 100% of the benefit is payable on the failure of 3 or more ADLs

then we could value the benefits as the following **separate policies:**

(i) 50% of the benefit on **the failure of 2 or more ADLs**

(ii) 50% of the benefit on **the failure of 3 or more ADLs**

Note: This would then require the estimation of:

$$i_{x+t}^{(2 \text{ ADLs})} \qquad {}_tp_x^{(<2 \text{ ADLs})} \qquad {}_tp_{x,d}^{(\geq 2 \text{ ADLs})}$$

$$i_{x+t}^{(3 \text{ ADLs})} \qquad {}_tp_x^{(<3 \text{ ADLs})} \qquad {}_tp_{x,d}^{(\geq 3 \text{ ADLs})}$$

for all **ages and deferred periods of interest.**

Advantages of the multiple-state model approach

- models the underlying process, hence **can be easily adapted to many product designs**

- parameters (transition intensities) are 'well defined' with **a simple form for their maximum likelihood estimates**

- no need to make **simplifying assumptions about the underlying process**

Disadvantages of the multiple-state model approach

- Data is required at the individual level of movement between the states of interest.

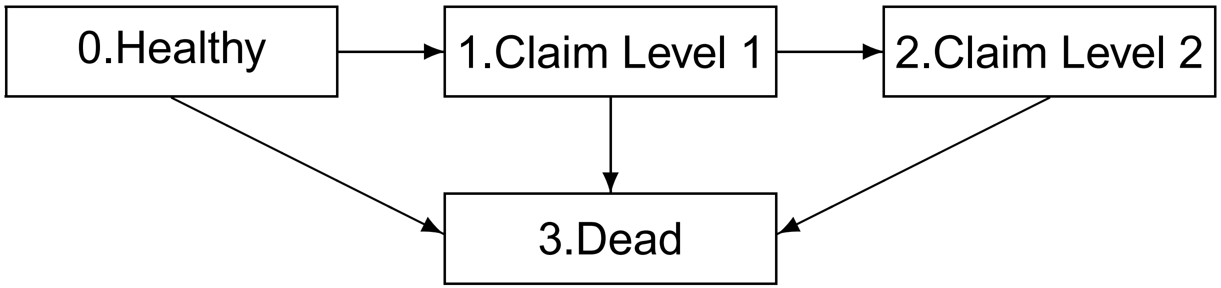
Note: In using a Markov model we are assuming that duration of disability **does not affect future transitions.**

Given sufficient data the appropriateness of this assumption could be investigated and**, if justified, a semi-Markov model could be used.**

Example

(Faculty and Institute of Actuaries: Subject 105, September 2003: Question 14. Note that this question is formulated in discrete time rather than continuous time just because it is a pencil-and-paper exercise to be completed under exam conditions.)

A life insurance company uses the following multiple-state model for pricing and valuing annual premium long-term care contracts, which are sold to lives that are healthy at outset.



Under each contract, the life company will pay the costs of long-term care while the policyholder satisfies the conditions for payment. These conditions are assessed every year on the policy anniversary, just before payment of the premium then due. If the policyholder satisfies the conditions, the annual amount of the benefit payable is paid immediately. A maximum of four benefit payments may be made under the policy, after which time the policy expires. The policy also expires on earlier death.

Premiums are payable annually in advance under the policy until expiry, and are waived if a benefit is being paid at a policy anniversary.

For lives at claim level 1, benefits of 60% of the maximum level are paid, while lives at claim level 2 receive 100% of the maximum level. The current maximum level is GBP 50,000 per annum and is expected to increase by 6% per annum compound in the future.

p_x^{ij} is the probability that a life aged x in state i will be in state j at age $x+1$ and the insurer uses the following probabilities for all values of x:

$$p_x^{00} = 0.87 \quad p_x^{01} = 0.1 \quad p_x^{02} = 0.0$$
$$p_x^{11} = 0.6 \quad p_x^{12} = 0.3 \quad p_x^{22} = 0.6$$

(i) Calculate the annual premium under the contract.

Basis:	Interest:	6% per annum
	Expenses:	7.5% of each permium

(ii) A policyholder has already received two benefit payments at level 1, and is about to receive a third benefit instalment. Calculate the reserves the office should hold for this policy immediately after the benefit payment is made, if the policyholder is assessed as entitled to either:

(a) benefit at level 1 = GBP 42,000 per annum

(b) benefit at level 2 = GBP 70,000 per annum

Reserve	Transition	
Basis:	Probabilities:	as given
	Interest:	5% per annum
	Benefit Inflation:	Inflation of the maximum benefit level of 7% per annum

Solution: (i) Let P = Annual Premium and SA = Sum assured.

$$\text{E}\left[\text{Income}\right] = P\sum_{t=0}^{\infty} v^t \,{}_tp_x^{00}$$

$$= P\sum_{t=0}^{\infty}\left[\frac{0.87}{1.06}\right]^t$$

$$= \frac{P}{1 - \frac{0.87}{1.06}} = 5.578947P$$

<u>Benefits</u>

Assume benefits are just about to begin. Since benefits escalate at the same rate as the discount rate, ignore interest.

$$1^{st} \text{ Payment} = 0.6 \times SA$$

$$2^{nd} \text{ Payment} = \begin{cases} 0.6 \times SA & \text{with prob } = 0.6 \\ SA & \text{with prob } = 0.3 \end{cases}$$

$$3^{rd} \text{ Payment} = \begin{cases} 0.6 \times SA & \text{with prob } = 0.6^2 \\ SA & \text{with prob } = 0.36^* \end{cases}$$

$(^{*}\text{Prob} = 0.6 \times 0.3 + 0.3 \times 0.6)$

$$4^{th} \text{ Payment} = \begin{cases} 0.6 \times SA & \text{with prob } = 0.6^3 \\ SA & \text{with prob } = 0.324^{**} \end{cases}$$

$(^{**}\text{Prob} = 0.3 \times 0.6^2 + 0.6 \times 0.3 \times 0.6 + 0.6^2 \times 0.3)$

Hence:

$$\begin{aligned} \text{EPV} &= SA\,[0.6\,(1 + 0.6 + 0.36 + 0.216) \\ &\qquad + (0.3 + 0.36 + 0.324)] \\ &= 114,480 \end{aligned}$$

Now we have that:

$$\text{P}[\text{claim starts at time } t] = {}_{t-1}p_x^{00}\, p_{x+t-1}^{01} = (0.87)^{t-1}\,(0.1)$$

Hence:

$$\begin{aligned} \text{E}\left[\text{Benefits}\right] &= 114,480 \sum_{t=1}^{\infty} (0.87)^{t-1}\,(0.1) \\ &= 114,480 \frac{0.1}{1-0.87} = 88,061.54 \end{aligned}$$

Therefore, the equation of value is:

$$0.925\,(5.578947\,P) = 88,061.54$$

$$\implies P = 17,064.43$$

(ii)(a) If the 3^{rd} instalment is at level 1, then the 4^{th} claim will be at level 1 with probability 0.6, or at level 2 with probability 0.3.

Interest and inflation no longer cancel, hence:

$$\begin{aligned} \text{Reserve} &= 42,000 \left(\frac{1.07}{1.05}\right) 0.6 + \frac{42,000}{0.6} \left(\frac{1.07}{1.05}\right) 0.3 \\ &= 47,080 \end{aligned}$$

(b) If the 3^{rd} instalment is at level 2, then the 4^{th} can only be at level 2, and will occur with probability 0.6.

$$\text{Reserve} = 70,000\left(\frac{1.07}{1.05}\right)0.6 = 42,800$$