Chapter 3: United-linked Policies

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Due to increasingly keen competition in insurance markets, there is a proliferation of various insurance products in order to cater for different financial needs of the clients. Various insurance products, which have more complicated contractual features than the traditional insurance products, like life insurance contracts and life annuities, have been innovated and developed in the insurance markets. One of the main features of these insurance products is that the benefits of the contracts depend on the returns or performance from certain investment funds. An unit-linked contract is a typical example of these types of contracts. It is very popular in the insurance markets. The contractual features of an unit-linked contract are relatively complicated compared with the traditional insurance products. In this case, it is difficult to use the traditional actuarial approach for evaluating the premiums and the reserves of these contracts. Instead, profit testing is a plausible and popular approach for evaluating premiums and the

reserves of unit-linked contracts. In this chapter, we will discuss the contract design of unit-linked contracts and the use of profit testing for evaluating the premiums and the reserves of the unit-linked contracts. The outline of this chapter is listed as follows:

Outline of Chapter 3

- Section 3.1: An introduction to unit-linked business and contract design

- Section 3.2: Profit testing of unit-linked contracts

- Section 3.3: Unit-linked reserves

- References: Course Note, Volume 4, Unit 5, Section 3, Pages 14-30

Section 3.1: An introduction to unit-linked business and contract design

- Two types of conventional policies
  - Non-profit: Policyholders pay **Fixed Premiums** and receives a **Known Benefits**
  - With-profit: Policyholders pay **Fixed Premiums** and receives an **Unknown Benefits**. The insurers make a profit if premiums and interest are **Large Enough** to pay for benefits and expenses

- Unit-linked policies
  - Policyholders pay **Variable Premiums**
  - The premiums are converted to **units**

- Charges are deducted
- Policyholders receive the **Unknown** value of the units
- Policyholders also receive a **Guaranteed** payment on **Death**
- The insurer makes a profit if charges are **Large Enough** to pay for expenses and death guarantees

- Unit-linked policies or funds in practice
  - The assets of all the investors are **Pooled** into a single fund
  - Each investor's share of the fund can be divided into units
    1. $V(t)$: The value of **All** investments in fund at time t

2. $N(t)$: The number of units held by **All** investors at time t

3. $u(t)$: The unit price at time t

4. Then,

$$V(t) = N(t) \times u(t)$$

– If an investor wants to invest X into the fund at time t, n new units will be created, where

$$n = \frac{X}{u(t)}$$

– Similarly, for withdrawals

$$\begin{aligned} V(t_+) &= V(t_-) + X \\ &= [N(t_-) + n]u(t) \\ &= N(t_+)u(t) \ , \end{aligned}$$

where both X and n are negative

– The unit value changes to reflect the return earned on the fund

$$u(t) = u(t-1)(1+i_t) \ ,$$

where i_t = Investment return from $t-1$ to t

– Example:

t	Unit Price	Cash Flow	New Units	Total Units	Total Value
0	2.0	100	50	50	100
1	2.5	50	20	70	175
2	3.0	−90	−30	40	120
3	2.5	0	0	40	100

1. Charges are ignored

2. Since we often invest in **Risky** assets, the price of units can go either up or down

– The units belong to the policyholder

  - The insurer will deduct charges to pay for expenses, guarantees (e.g. on death) and profit

- Different types of charges
  - Fixed monetary amounts (e.g £50)
  - Percentage of premium
  - Percentage of the investor's fund

- Goal: Try to **match** costs and charges

- Bid/Offer Spread
  - Percentage of premium
  - Offer price: Price policyholders pay to buy one unit

- Bid price: Price policyholders receive when selling one unit
- There is a bid/offer spread of λ if

$$\frac{\text{Bid Price}}{\text{Offer Price}} = 1 - \lambda$$

- The bid value of units bought $= (1 - \lambda)a_t P_t$
- Charge $= \lambda a_t P_t$
- Typically, $\lambda = 5\%$
- This charge is similar to the bid/offer spread in offer assets (e.g. equities)

- Allocation Rate
  - Percentage of premium
  - Only a proportion a_t of each premium P_t is used to buy units at the **Offer** price

- Offer value of units bought $= a_t P_t$
- Charge $= (1 - a_t) P_t$
- a_t is called the allocation rate
- Early years: $a_t = 0\% \rightarrow 90\%$
- Later years: $a_t = 95\% \rightarrow 105\%$
- This charge approximately matches most of the initial and renewal expenses and comission

- Policy fee
  - Monetary amount
  - The policy fee is deducted from the policyholder fund on a regular basis (e.g. monthly or annually)

- Units are sold to pay the fee
- A typical fee = £25 per annum
- Example: Suppose the policy fee is £25 and the bid price is £2.50. Then, sell 10 units
- The charge should be similar to the insurer's fixed costs per policy

• Fund Management Charge (FMC)

- Percentage of the fund
- A percentage deduction from the **Bid** value of the units
- The charge covers the investment expenses of the insurer
- In practice, the charge is deducted daily by **Reducing** the **Bid** price

- Typical FMC = 1% per annum
- To make life easy, assume the FMC is deducted from the unit fund at the **End** of each year, just before the payment of benefits

- Mortality Charges
  - Monetary amount
  - On death, the policyholder receives the maximum of the value of the unit fund F_t and a guaranteed minimum payment S_t
  - Example: Death benefit = $\max(S_t, F_t) = F_t + \max(0, S_t - F_t)$

- Since the insurer must pay for the additional benefit above the unit fund, the insurer charges all policies for its **Expected Death Strain**

- Mortality Charge (MC) $= q_{x+t-1}(S_t - F_t)$ if $S_t > F_t$; MC $= 0$ if $S_t \leq F_t$

- If the fund grows through time, MC decreases to zero

- In practice, MC is deducted every month

- For simplicity, assume MC is deducted at the end of the year, **After The FMC**

- Some exam questions have **No** MC (i.e. They have a lower allocation rate to compensate)

- Surrender Penalty
  - Percentage of fund
  - The charges in the early years of the contract are often too small to cover the high inital expenses
  - However, profits are made in the later years
  - Hence, on surrender, the insurer makes a charge to cover the future profits it would have made
  - Example:

YR	Surrender Penalty
1	50% $\times$ Fund
2	30%
3	10%
≥ 4	0%

- Policy features: Conventional non-profit
  - Advantages:
    1. Designed for protection/insurance
    2. Guaranteed benefits
    3. No investment risk for policyholder
    4. Easy to understand
  - Disadvantages:
    1. Investment risk for insurer $\Longrightarrow$ Invest in safe assets
    2. Low returns
    3. Not flexible: Alternations are difficult
    4. Expenses hidden from policyholder: Premium loading

- Policy features: Unit-linked
  - Advantages:
    1. Designed for saving and investment
    2. No investment risk for insurer
    3. Policyholder can choose investments and will or may invest in risky assets
    4. High expected returns
    5. Flexible: Alternations are easy
    6. Expenses are explicit to policyholder
  - Disadvantages:
    1. No guaranteed maturity value
    2. Investment risk for policyholder
    3. Confusing large number of charges

Section 3.2: Profit-testing of unit-linked

- Objectives for conventional business:
  - **Set Premium**
  - To meet our profit criteria under the premium basis
- Objectives for unit-linked:
  - **Set Charges**
  - To meet our profit criteria under the **Premium Basis**
  - Need to know the insurer's profit signature
  - First need to find the value of the policyholder's assets at each year end

- The Unit Fund
  - The policyholders investment = Number of units they hold
  - Call the policyholder's assets the **Unit Fund**

- Examples:
  - Five-year unit-linked endowment assurance
  - Premium = $£5,000$ per annum, annually in advance
  - Survival Benefit: Bid value of units at the end of the term
  - Death Benefit
    1. Paid at the end of year of death

2. The maximum of $\pounds 20,000$ and the bid value of units

3. Paid after deduction of FMC and MC

– Charges:

1. Allocation Rate: 70% in year one and 102%, otherwise

2. Bid/Offer Spread: 5%

3. Policy Fee: £30 deducted from units at start of each year

4. FMC: 1% taken at the end of the year

5. Death Charge: 1%$\times$ Sum at risk at the end of year, after FMC

6. Unit growth rate: 8%

– Determine the year-end fund after charge for each year

– Solution: Discuss in Lecture!

– Mortality benefit in year 5 = 27,013.20

– Death Benefit

YR	Amount	Unit Fund	+	Insurer
1	20,000	3,357	+	16,643
2	20,000	8,623	+	11,377
3	20,000	14,311	+	5,689
4	20,450	20,450	+	0
5	27,013	27,013	+	0

- Surrender Benefit

YR	Unit Fund	Penalty	Payout
1	3,357	50%	1,678
2	8,623	30%	6,036
3	14,311	10%	12,880
4	20,450	0%	20,450
5	27,013	0%	27,013

- Sterling Fund
  - Given the **Policyholder's** expected cashflows, we can calculate the **Insurer's** expected cashflows
  - Use the same convention as before
  - Calculate the expected cashflow per policy in-force at the start of the year

– The cashflows:

1. Charges + Interest - Expenses - Benefits

2. Denominated in pounds sterling or cash rather than units

– The **Sterling Cashflows**, SCF_t, are paid into the insurer's sterling fund

– Assume SCF_t is transferred to capital at the end of year t

- Example:
  - Experience Basis
  - Expenses

YR	
1	$40\% \times \text{PREM}$
2	$10\% \times \text{PREM}$
3, 4, 5	$2.5\% \times \text{PREM} + £20$

  - Note: Expenses may be different from charges
  - Mortality: 1% per annum
  - Note: Experience may be different from the charges
  - Interest: 4% per annum on sterling fund
  - Note: Sterling and unit funds are invested in different assets

– Calculate the expected cashflow per policy in-force at the start of each year

– Solution: Discuss in Lecture!

Section 3.3: Unit-linked Reserves

- Motivation: Since we have negative sterling cashflows after year one, we need to set up reserves

- Actuarial Formulae: Too complicated to be used for calculating reserves for unit-linked policies

- Use profit test to calculate reserves

- Re-calculate the profit test using a **Cautious** experience basis which we call a **Valuation Basis**

- Example:
  - Valuation Basis
  - Expenses

YR	
1	$40\% \times \text{PREM}$
2	$10\% \times \text{PREM}$
$3, 4, 5$	$2.5\% \times \text{PREM} + £40$

  - Mortality: 2% per annum
  - Interest: $i_u = 6\%$ and $i_s = 3\%$
  - Note: The premiums and charges are fixed
  - Demostration: Discuss in Lecture!

- Reserving Algorithm:
  - Basic idea
    1. Make negative cashflows become zero
    2. Recall: $\text{PRO}_t = \text{SCF}_t + {}_{t-1}V(1+i_s) - p_{x+t-1} \times {}_tV$
  - Step I: Calculate SCF_t with no reserves
  - Step II:
    1. Let m denote the greatest value of t for which $\text{SCF}_t < 0$
    2. Set ${}_tV = 0$, for all $t \geq m$ (No need for reserves since $\text{SCF}_t \geq 0$, for all $t > m$)

– Step III:

1. Set ${}_{m-1}V$ such that $\text{PRO}'_m = 0$
2. $\text{PRO}'_m = \text{SCF}_m + {}_{m-1}V(1 + i_s)$
3. ${}_{m-1}V = -\frac{\text{SCF}_m}{1+i_s} > 0$

– Step IV: Check the effect of setting up the reserve ${}_{m-1}V$ on PRO'_{m-1}. Assume ${}_{m-2}V = 0$. Then,

$\text{PRO}'_{m-1} = \text{SCF}_{m-1} - p_{x+m-2} \times {}_{m-1}V$

1. If $\text{PRO}'_{m-1} \geq 0$, set ${}_{m-2}V = 0$ and search for $t < m-1$ such that $\text{PRO}'_t < 0$ and repeat the above process
2. If $\text{PRO}'_{m-1} < 0$, set ${}_{m-2}V$ such that

$$\begin{aligned} 0 &= \text{PRO}''_{m-1} \\ &= SCF_{m-1} - p_{x+m-2} \times {}_{m-1}V + (1 + i_s)_{m-2}V \\ &= \text{PRO}'_{m-1} + (1 + i_s)_{m-2}V \end{aligned}$$

This implies that

$$_{m-2}V = -\frac{\text{PRO}'_{m-1}}{1+i_s} > 0$$

Then, repeat Step IV for PRO'_{m-2}

– Step V:

1. Now, we have $\text{PRO}''_t \geq 0$, for all $t > 1$ as required

2. PRO'_1 may be negative requiring an injection of capital

3. Then, we refer to $-\text{PRO}_t$ as the **New Business Strain**

- Example:

t	SCF_t
1	-100
2	10
3	-30
4	80
5	80
6	-40
7	50
:	:

  - Assume $i_s = 4\%$ and Mortality $= 10\%$ (i.e. $p_x = 0.9$)
  - ${}_5V = -\frac{\text{SCF}_6}{1.04} = 38.46$ (i.e. $\text{PRO}_6' = 0$)
  - Then,

$$\begin{aligned} \text{PRO}_5' &= \text{SCF}_5 - p_{x+4} \times {}_5V \\ &= 80 - 0.9 \times 38.46 \\ &= 45.39 > 0 \end{aligned}$$

  - ${}_4V = 0$
  - Since $\text{PRO}_3' < 0$, set ${}_3V = 0$

- ${}_2V = -\frac{\text{SCF}_3}{1.04} = 28.85$

- Then,

$$\begin{aligned} \text{PRO}_2^{'} &= \text{SCF}_2 - p_{x+1} \times {}_2V \\ &= 10 - 0.9 \times 28.85 \\ &= -15.97 < 0 \end{aligned}$$

- ${}_1V = -\frac{\text{PRO}_2^{'}}{1.04} = 15.36$

- ${}_tV = (15.36, 28.85, 0, 0, 38.46, 0, 0, \dots)$

- Then,

$$\begin{aligned} & \text{PRO}_t^{''} \\ = \; & (-113.82, 0, 0, 80, 45.39, 0, 50, \dots) \end{aligned}$$

- Compare

$$\begin{aligned} & \text{SCF}_t \\ = \; & (-100, 10, -30, 80, 80, -40, 50, \dots) \end{aligned}$$

- Consider the last example of sterling fund:
  - Note that

$$\begin{array}{rl} & \mathrm{SCF}_t \\ = & (-437.02, -354.63, 98.53, 213.12, 278.60) \end{array}$$

  - ${}_5V = {}_4V = {}_3V = {}_2V = 0$
  - Then,

$${}_1V = -\frac{\mathrm{SCF}_2}{1+i_5} = \frac{354.63}{1.03} = 344.30$$

  - Given the reserves, investigate the effect on the original experience basis

- Remarks:
  - After setting up reserves, calculate the profit signature σ_t
  - Then, we measure the profit in the same way as conventional policies (e.g. RDR, DPP, IRR)

- Example:
  RDR = 10 %
  Expected profit = 27.45
  DPP = 5 years
  IRR = 11.94%

- Dealing with initial expenses
  - Initial expenses are very large (i.e. New Business Strain)
  - Set charges to recover the strain as quickly as possible
  - Reduced initial allocation (or Front-end load): Allocate less to units in the early years then later years

- Example:
  - Use a partial front end load with allocation rates

Yr	Allocation
1	70%
$2-5$	102%

  - Then,

$$\begin{aligned} SCF_1 &= -271 \\ SCF_2 &= -239 \end{aligned}$$

  - Use a full front end load with allocation rates

Yr	Allocation	SCF
1	65%	-27
2	97%	2
$3-5$	105%	

- Advantages of Front-End Load:
  - Simple
  - Can match expenses very closely
  - Remove strain: Increase profits or reduce charges
- Disadvantages of Front-End Load:
  - Make it clear to the policyholder that his/her investment is worth very little in the early years
- Increased FMC
  - Recover the initial expenses by a higher FMC

- Common approach: Sell two different types of units, namely Capital Unit Fund and Accumulation Unit Fund

- Use premiums in the early years to buy **Capital** (initial) units which have a higher FMC (e.g. 5%)

- An important note: Determine the FMC based on the year in which the premium is invested, but **Not** the year the charge is deducted

- Use premiums in the later years to buy **Accumulation** units which have a low FMC (e.g. 1%)

- Example:

  - Allocation rate = 102% **All** years

Yr	Premiums	FMC
1, 2	Capital	5%
3, 4, 5	Accumulation	1%

- Invest the units in the same assets (i.e. Both fund earns 8% per annum)

- Mortality Charge = (20,000 - Capital Value - Accumulated Value) $\times(\frac{0.01}{1-0.01})$

- Suppose mortality charge is deducted from the capital unit fund.

- Assume, further, that policy fee is deducted from the same fund as the premiums are paid into

- Demostration: Discuss in Lecture!

- Sterling Fund

  1. Premium = 5,000

  2. Allocation = 102%

  3. Bid/Offer = 5%

4. Allocated to policyholder at bid

$$\begin{aligned} &= 5,000 \times 102\% \times 95\% \\ &= 4,845 \end{aligned}$$

5. Expenses = 2,000 in year one

6. Policy fee = 30

Sign	Income in year one	Item
+	5,000	Premium
−	4,845	Allocated to policy holder
+	30	Policy fee
−	2,000	Office expenses
−	1,815	Fund at start
−	73	Interest @ 4%
+	260	FMC at 5%
+	152	Mortality charge to policyholder
−	152	Mortality cost to office
−	1,628	SCF_1

- Disadvantages of Increased FMC
  - Future charges depend on unit growth
  - Does not match expenses
    1. Charge is too small in early years
    2. Charge is too large in later years
  - More complex than front end load
  - Conceals charges from policyholders

- Solution: Actuarial Funding

End of Chapter 3