Maple Workshops F7.1SC3, 2007
Tutorial 2 (Week 3)

This worksheet is intended to help you investigate the equation solving and linear algebra features of Maple. The notes which have been handed out and the 'on-line' help should assist you to work through the sheet.
Get into good habits: **layout your worksheet neatly and clearly number each answer**. The assessment next week will be made up of a few questions similar to examples on this sheet!

1. Convert the following expression to partial fractions:

$$\frac{x^2 - 6x + 9}{(x^2 - 9)^2 + (x - 3)^3}$$

   (`convert`).

2. Set `f` as $(x + y + 4)^3/(y^2 - 16)$, then substitute $x - 4$ for y using the `subs` command and finally `factor` the result.

3. Use `solve` to solve the equation

$$11a + \frac{4}{b} = 4$$

   for b and then use `subs` to compute the solution when $a = 1/4$.

4. Find the solution of the set of equations

$$\begin{aligned} 5x + 6y &= z \\ 3z + 4y &= 7 \\ z + 5x &= 4 - y \ . \end{aligned}$$

5. Determine the general solution for x and y of the pair of simultaneous equations

$$\begin{aligned} 2x - y &= 2 \\ x + 2y &= c \ . \end{aligned}$$

   Compute the solutions when $c = 0$, and when $c = -3/2$ (`solve, subs`).

6. First assign and then use `plot` to graph the function

$$x^3 - 2x^2 - 7x + 1 + \sin(x)$$

   for $-3 \leq x \leq 4$. With the help of the plot use `fsolve` to find *all of the roots* of the function.

7. Use `fsolve` to find the solutions of the equation

$$\ln(1 + x^2) = \cos(x) \ .$$

   [Plot the functions to ensure you have all the solutions.]

cont.

8. Assign and then work out the determinant of the matrix

$$M = \begin{pmatrix} 1 & a & b \\ 1 & a^2 & b^2 \\ 1 & a^3 & b^3 \end{pmatrix}.$$

Factor the result and deduce the solutions to $\det(M) = 0$ (`with(linalg), matrix, det`).

9. Assign the matrix

$$A = \begin{pmatrix} 3 & 4 & -5 \\ 5 & 4 & 3 \\ 1 & 0 & -1 \end{pmatrix}.$$

(a) Find A^4, using `evalm` to display the matrix result.

(b) Work out the determinants of A and A^4.

10. Using A from the previous question and

$$B = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 2 & -2 \\ 3 & 3 & 0 \end{pmatrix}$$

work out the products AB and BA, hence demonstrating that matrix multiplication does not commute in general. Evaluate the determinant of these two products and also $\det(A)\det(B)$, interpreting your results.

11. Let

$$C = \begin{pmatrix} 3 & 2 \\ -1 & 9 \\ 0 & 4 \end{pmatrix},$$

work out the transpose of C (denoted C^{T}) and the products CC^{T} and $C^{\mathrm{T}}C$ (`transpose`).

12. Use B from above and let

$$\underline{d} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Solve $B\underline{x} = \underline{d}$ for $\underline{x}$ (`linsolve` or `inverse`).

13. Write the system of linear equations

$$\begin{aligned} 2x - 2y - 3z &= 1 \\ 4x - y - z &= 6 \\ 2x + y + 3z &= 4 \end{aligned}$$

in matrix form and hence find x, y and z without using `solve`.

14. Use the `->` notation to define Maple functions for

$$f(x) = x^2\exp(-x)\ , \quad g(x) = \tan(x^2).$$

As an example that order matters when composing functions, evaluate $f(g(1))$ and $g(f(1))$ to 3 significant figures.