F7.1SC3 Tutorial 1 - Solutions

Question 1

```
> 5/7+19/23-286/403;
```

$$\frac{4146}{4991}$$

```
> evalf(%,8);
```

$$0.83069525$$

Question 2

```
> y1:=2^(15)/3^(11);y2:=exp(Pi*sqrt(42)/3);
```

$$y1 := \frac{32768}{177147}$$

$$y2 := \mathbf{e}^{\left(\frac{\pi\sqrt{42}}{3}\right)}$$

```
> evalf(y1,10);evalf(y1,20);
```

$$0.1849763191$$

$$0.18497631910221454498$$

```
> evalf(y2,10);evalf(y2,20);
```

$$885.9103754$$

$$885.91037562840944067$$

Question 3

```
> factor(x^3+x^7-2*x^5);
```

$$x^3 (x-1)^2 (x+1)^2$$

Question 4
Part(a)

```
> 5!*8*9/(2!*3!);
```

$$720$$

Part(b)

```
> expand((2*x+3)^5);
```

$$32\,x^5 + 240\,x^4 + 720\,x^3 + 1080\,x^2 + 810\,x + 243$$

```
> # the coefficient of x^3 is 720, just as found in part (a)
```

Question 5

```
> y3:=6*x^3+25*x^2+21*x-10;
```

$$y3 := 6\,x^3 + 25\,x^2 + 21\,x - 10$$

```
> solve(y3=0,x);
```

$$-2, \frac{1}{3}, \frac{-5}{2}$$

```
> factor(y3);
```

$$(2x+5)(3x-1)(x+2)$$

```
> # solutions of y3=0 are 2x+5=0 (i.e. x=-5/2), 3x-1=0 (x=1/3), and
  x+2=0 (x=-2). Thus we find the same solutions using either method
  as expected.
```

Question 6

```
> expand((2*x^3+(1/(3*x)))^(12));
```

$$\frac{1760}{19683} + 4096x^{36} + 8192x^{32} + \frac{22528x^{28}}{3} + \frac{112640x^{24}}{27} + \frac{14080x^{20}}{9} + \frac{11264x^{16}}{27} + \frac{19712x^{12}}{243} + \frac{2816x^{8}}{243} + \frac{880x^{4}}{729} + \frac{88}{19683x^{4}} + \frac{8}{59049x^{8}} + \frac{1}{531441x^{12}}$$

```
> # the term in x^8 has coefficient 2816/243, and the term
  independent of x is 1760/19683
```

Question 7

```
> (1-I)*(3+7*I);
```

$$10 + 4I$$

```
> (6-3*I)/(4-I);
```

$$\frac{27}{17} - \frac{6}{17}I$$

Question 8

```
> exp(3*I*Pi/4)/exp(I*Pi/4);
```

$$\frac{-\frac{\sqrt{2}}{2} + \frac{1}{2}I\sqrt{2}}{\frac{\sqrt{2}}{2} + \frac{1}{2}I\sqrt{2}}$$

```
> simplify(%);
```

$$I$$

```
> # simplifying the original expression by hand gives exp(I*Pi/2),
  the complex number with modulus 1 and argument Pi/2 ... in other
  words I
```

Question 9

```
> sqrt(6-7*I);
```

$$\sqrt{6-7I}$$

```
> # need to encourage Maple to give us a numerical answer
> evalf(sqrt(6-7*I),4);
```

$$2.759 - 1.269\, I$$

```
> # this is one root, the other is obtained by multiplying by -1.
  Hence the other solution is
> %*(-1);
```

$$-2.759 + 1.269\, I$$

Question 10
Part(a)

```
> z:=2+3*I;w:=5+I;
```

$$z := 2 + 3\, I$$
$$w := 5 + I$$

```
> z*w;
```

$$7 + 17\, I$$

```
> evalf(argument(z*w));
```

$$1.180189283$$

```
> evalf(argument(z)+argument(w));
```

$$1.180189283$$

```
> # the two answers agree!
```

$$0.$$

Part(b)

```
> z:=-1+sqrt(3)*I; w:=-sqrt(3)+I;
```

$$z := -1 + \sqrt{3}\, I$$
$$w := -\sqrt{3} + I$$

```
> z*w;
```

$$(-1 + \sqrt{3}\, I)\,(-\sqrt{3} + I)$$

```
> simplify(%);
```

$$-4\, I$$

```
> evalf(argument(z*w));
```

$$-1.570796327$$

```
> evalf(argument(z)+argument(w));
```

$$4.712388981$$

```
> # we have calculated the principal argument of z*w which by
  definition must lie in the range from -Pi to Pi. arg(z)+arg(w) is
  clearly outside this range, but is equal to -1.570.. mod 2Pi (in
  particular arg(z*w)=arg(z)+arg(w)-2*Pi in this example).
```

Question 11

```
> z:='z';  # unassign z from previous question
```

$$z := z$$

```
> sols:=solve(z^3=8,z);
```

$$sols := 2, -1 + \sqrt{3}\, I, -1 - \sqrt{3}\, I$$

```
> # three solutions z=2, z=-1+sqrt(3)*I, and z=-1-sqrt(3)*I
> argument(sols[1]); abs(sols[1]);
```

$$0$$

$$2$$

```
> argument(sols[2]); abs(sols[2]);
```

$$\frac{2\pi}{3}$$

$$2$$

```
> argument(sols[3]); abs(sols[3]);
```

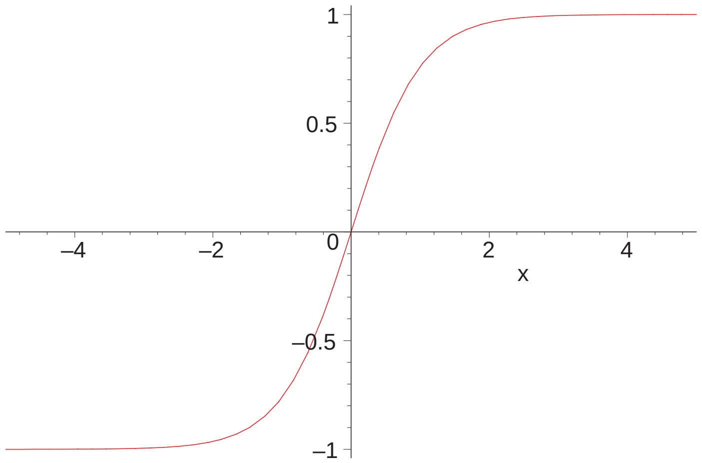
$$-\frac{2\pi}{3}$$

$$2$$

```
> # all three solutions have modulus two, and are evenly spaced on a
  circle of radius 2 centred at the origin of the argand diagram
```

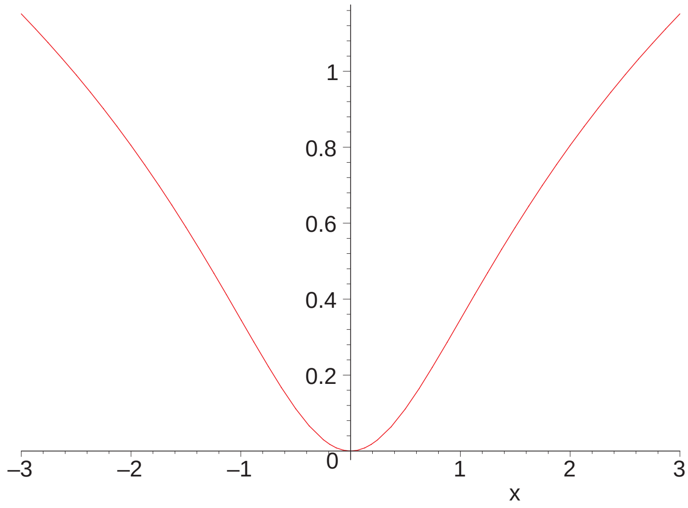
Question 12
Part(a)

```
> plot(tanh(x),x=-5..5);
```



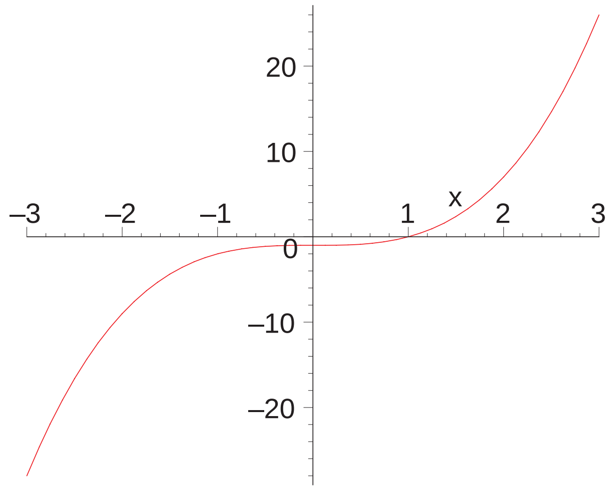
Part(b)

```
> plot(ln(sqrt(1+x^2)),x=-3..3);
```



Question 13
Part(a)

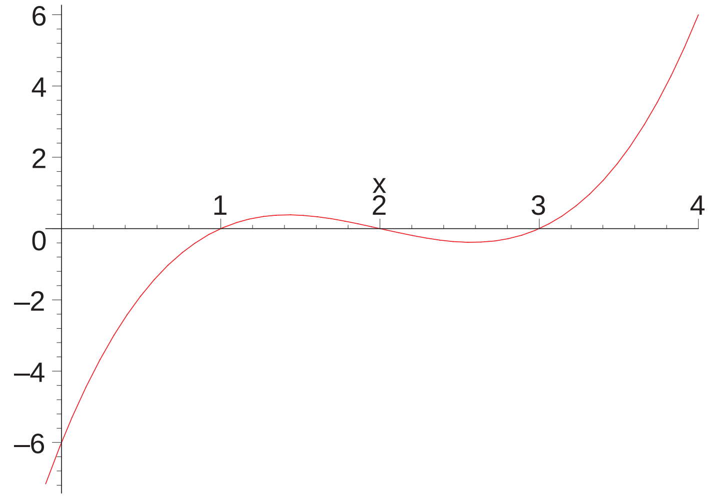
```
> plot(x^3-1,x=-3..3);
```




```
> # from the plot there is one real root (x=1), and hence there are
  two complex roots
```

Part(b)

```
> plot(x^3-6*x^2+11*x-6,x=-0.1..4);    # choose plot limits by trial
  and error
```



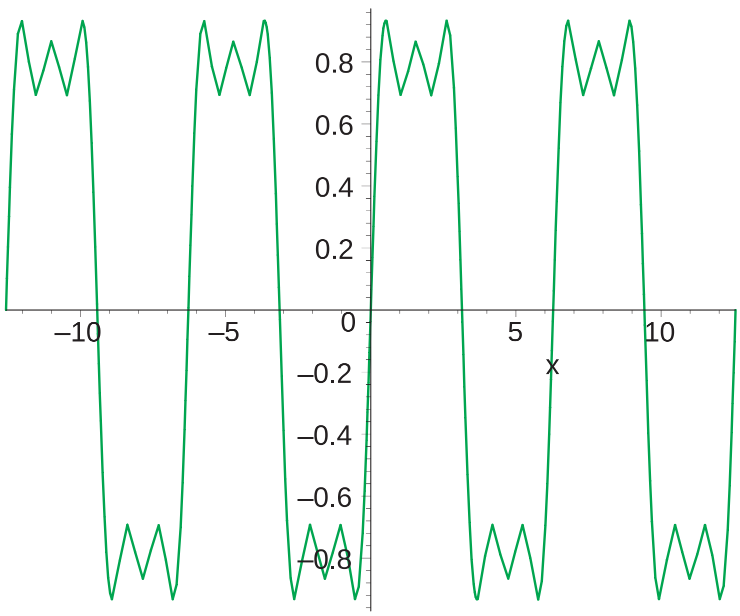

```
> # in this case all three roots are real (x=1,x=2,x=3)
```

Question 14

```
> y:=sin(x)+(1/3)*sin(3*x)+(1/5)*sin(5*x);
```

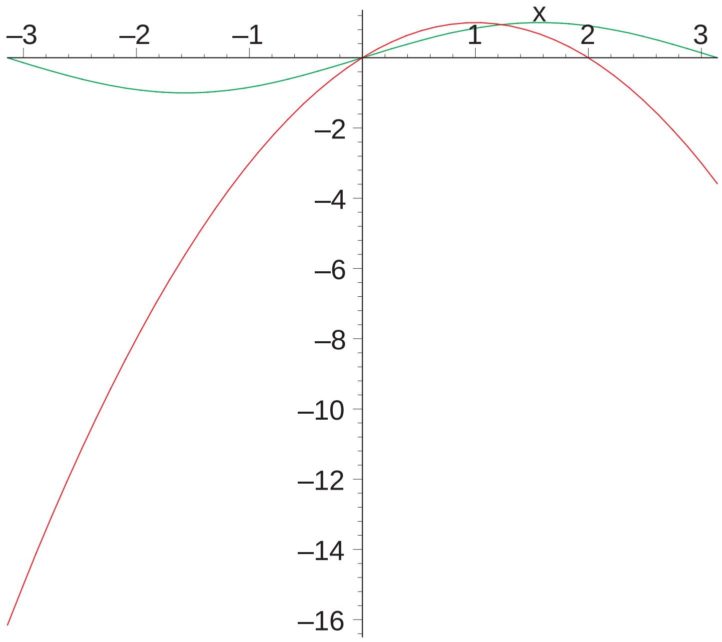
$$y := \sin(x) + \frac{1}{3}\sin(3\,x) + \frac{1}{5}\sin(5\,x)$$

```
> plot(y,x=-4*Pi..4*Pi,color=green,thickness=3);
```



Question 15

```
> plot({sin(x),x*(2-x)},x=-Pi..Pi);
```




```
> # from the plot there are two solutions of sin(x)=x(2-x), one at
  x=0, and the other at approximately x=1.2  Later on we will learn
  the fsolve command :
> fsolve(sin(x)=x*(2-x),x,1..2);
```

1.235783561

```
> # hence the second solution is at x=1.2357...
```