Maple Workshops F7.1SC3, 2007 Assessment 3 (Week 7)

Try all questions on the assessment. At the end of the class print out your worksheet and hand it in. Layout your worksheet neatly and clearly number each answer.

> The worksheet you hand in at the end of the class should have the following information on it at the top: Maple Assessment 3 Your full name, Your department

1. Assign the function

$$f(x) = \left(\frac{3x^2 - x + 1}{2x^2 + x + 1}\right)^{\frac{x^3}{1 - x}}$$

(a) Find the value of

 $\lim_{x \to \infty} f(x)$

- (b) Calculate the first derivative of f, i.e. f'(x) and evaluate f'(2) to 5 significant figures.
- 2. (a) First state then evaluate the following integral to 7 significant figures

$$\int\limits_0^\pi e^{2x^{1/3}} \,\mathrm{d}x$$

- (b) Plot the curves $y = 5 x^2$ and $y = x + \sqrt{2}$ on a single graph and use Maple to determine where the functions cross. Thus calculate the area enclosed between the two curves, giving your answer to 8 significant figures.
- 3. Find the general solution of the second order differential equation for y(x) given by

$$x(1-x)y'' + 2xy' - 2y = 0$$

and then determine the specific solution which satisfies the boundary conditions

$$y(1) = 0$$
, $y'(2) = 3$.

Finally graph the solution for $0 \le x \le 3$.

4. Find the solution of the differential equation for g(t) given by

$$\ddot{g} - 2\dot{g} + g = \cos(2t)$$

together with the initial conditions

$$g(0) = 0$$
, $\dot{g}(0) = -1$.

Hence determine g(10) and g(100) evaluating up to 5 significant figures.

5. Solve the recurrence relation

$$u_{n+1} = u_n \left(\frac{2n+3}{2n+5}\right)^2 + \frac{1}{2n+5} ,$$

with the initial condition $u_0 = 2/3$.

Using your solution find the limit: $\lim_{n\to\infty} u_n$.