COUPLED COHERENT NONLINEAR OPTICAL SYSTEMS

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The physics behind coupling in coherent nonlinear optical structures used as all-optical switches is briefly reviewed, and the assumptions made in modelling these with coupled Duffing equations are outlined. Extending these to the assumptions pertaining to pulse propagation produces an assortment of coupled nonlinear Schrödinger equations.

The story so far ...

Light propagating through a waveguide can exchange between two states. These may be two directions of travel, two (or more) cores in the waveguide, two states of polarization, or two different modes. As input power is varied continuously, the fraction of power exchanged between the two states may vary discontinuously. This is the basis of all-optical switching.

If a signal's electric field **E** is composed of two components, whose amplitudes vary with distance of propagation z, it can be written

$$E = a_1(z)E_1 + a_2(z)E_2,$$

where E_j represents the known, constant electric field associated with each state, and is so normalized that it carries unit power. The power in state j is then $|a_j|^2$.

Propagation & Waveguiding

The starting point in analysing problems of light propagation is the set of Maxwell equations. For propagation through a dielectric material, these simplify to a wave equation:

$$\left[\nabla^2 - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right] E - P^{(NL)} = -\nabla(\frac{1}{n^2}E \cdot \nabla n^2). \tag{1}$$

The dielectric's linear refractive index is described by the variation

$$n^2(r,z) = n_0^2\left[1 + 2\Delta(g(r) + \epsilon h(r,z))\right], \tag{2}$$

with $g(r)$ being the waveguiding feature. $P^{(NL)}$ is discussed further below.

Assumption 1: In most waveguiding structures, the total variation in refractive index is small. This means $\Delta \ll 1$, so the term on the right of eq (1) can be ignored and the normal modes of the system are approximately TEM-modes, i.e. have electric & magnetic fields orthogonal to the direction of propagation and can be expressed as

$$E = \begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix} A(x,y)e^{i\beta z}e^{-i\omega t}.$$

It also means, correct to order Δ, that $\beta = \omega n_0/c$ with ω the angular frequency of the wave.

Assumption 2: The nonlinearity is weak, so that the modal field $\psi(x,y)$, i.e. the x, y variation of **E** , associated with the signal approximately equals that found for an associated linear eigenvalue problem:

$$\left[\nabla_T^2 - n^2\frac{\omega^2}{c^2}\right]\psi = \beta^2\psi. \tag{3}$$

To simplify analysis, the normalization of the modal field is chosen so that $\int_{R^2} dr\ \psi^2(r) = 1$. A further common, but unnecessary, assumption is that ψ is Gaussian in form.

Nonlinear Polarization

The form of the nonlinear polarization depends on the symmetries exhibited by the dielectric, and the frequencies present in the waveguide.

Most commonly considered is the cubic (Kerr) effect of isotropic materials which possess full inversion symmetry. $P^{(NL)}$ is given by a third-order nonlinear susceptibility tensor $\chi^{(3)}_{jklm}(-\omega_\alpha;\omega_\gamma,\omega_\delta,\omega_\zeta)$. The elements and frequency dependence can be greatly simplified. In summary, away from material resonances, i.e. absorbtion of power from the signal by the dielectric and Kleinmann symmetry prevails,

$$\frac{1}{\epsilon_0}P_x^{(NL)} = 3\chi_{xxxx}|E_x|^2E_x + 6\chi_{xxyy}|E_y|^2E_x + 3\chi_{xxyy}E_x^*E_y^2.$$

This is premised on the following:

Assumption 3: Only those frequencies initially launched in the waveguide are important. (This is not useful in frequency mixing problems.)

Anisotropic materials have a second-order nonlinearity. This means their nonlinearity is quadratic (Pöckels) in form, and waves of different frequencies are always combined.

Coupled Equations — CW case

In cw operation, all-optical switching of signals is governed by a set of coupled Duffing equations. These have the general form

$$\begin{aligned} ia_1' + C_1a_1 + Ke^{-i\zeta}e^{i2\Gamma}a_2 + D_1|a_1|^2a_1 + 2B|a_2|^2a_1 + Ea_1^*a_2^2 = 0 \\ ia_2' + C_2a_2 + Ke^{i\zeta}e^{-i2\Gamma}a_1 + D_2|a_2|^2a_2 + 2B|a_1|^2a_2 + Ea_2^*a_1^2 = 0 \end{aligned} \tag{5}$$

Values $C_j, D_j, B, E, K, \Gamma$ and ζ are constants, relevant to the particular coupled system under consideration. How are they found?

Bragg grating: For purposes of analysis, in-fibre gratings are assumed adiabatic, periodic perturbations of straight, single-cored waveguides. There is only one polarization present in such problems. The wavelength of the grating is defined by

$$\Lambda = 2\pi/|\beta_1 - \beta_2|$$

so that it permits the phase-matching of the two modes. It is emphasised that backward travelling modes correspond to negative values of β_j. Analysis assumes no energy is lost as radiation, and only the modes close to being phase matched are considered. Expansion of $\mathbf{E}$ in terms of the modes of the original waveguide, substitution into eqs (1) to (3), and subsequent Fourier decomposition gives the relevant coupled mode equations, namely eqs (5) with suitably defined constants. Some argument exists as to whether E effectively vanishes.

Rocking filter: In this case a periodic twist is inserted in the fibre, which is birefringent. Two polarization states exist in the waveguide, as a linearly polarized input is slowly rotated, first one way, then the other. They have distinct values β_1 & β_2. Formalism is similar to the Bragg grating.

Directional coupler: For purposes of most analysis, directional coupers are assumed straight, invariant waveguides. Recent work is designed to relax such assumptions. The waveguiding function g for the coupler is a linear sum of waveguiding functions g_j for the cores separately. There is only one polarization present in such problems.

Assumption 4: The mth mode of the composite cored waveguide (a "supermode") is a linear combination of the modes of the N cores in isolation from each other, i.e.

$$\psi^{(m)} = \sum_{j=0}^{N} q_j^{(m)} \psi_j .$$

This is substituted into eqs (1) to (3) as they pertain to each waveguide: g_j and g. The coupled mode equations follow. It is common to ignore the nonlinear coupling between the cores, i.e. $B = 0 = E$, as it is much weaker.

Solutions

Solutions to eqs (5) may be found in terms of Jacobi elliptic functions. Depending on the parameters of the physical system, it is possible to

(i) switch from a large fraction of power being exchanged between states, to a small fraction, as input power increases through some critical value;

(ii) switch from a small fraction, to a large fraction, as input power increases through some critical value;

(iii) have no switching but a continuous variation of the fraction exchanged, possibly increasing to 100%, as the input power varies.

What does this mean for problems of pulse propagation? In an attempt to answer, we consider first a single pulse, with carrier frequency ω_0.

Pulse propagation in a solitary state

A signal that varies in time has an associated spectrum which is merely a Fourier transform. As only one polarization component is present in the waveguide, the vector direction of the fields can be ignored. Hence, write the time variable electric field as

$$E = \frac{1}{2}\left[e^{i(\beta_0 z - \omega_0 t)}\int_{-\infty}^{\infty} d\nu \; e^{-i\nu(t-\beta_0' z)} S(z,\nu)\psi(r;\omega_0+\nu) + c.c.\right], \tag{6}$$

where subscript zeros indicate evaluation at the carrier frequency. Noting β_0' is the reciprocal of group velocity, define the shifted time by $\xi = t - \beta_0' z$, which is a co-ordinate moving with the centre of the pulse. Given this, it follows that

$$\begin{aligned} &S_{zz} + 2i(\beta_0 + \nu\beta_0')S_z + [\beta(\nu+\omega_0) - (\beta_0+\nu\beta_0')^2]S \\ =&\frac{(\omega_0+\nu)^2}{4c^2}[3\int_{R^2} + \int_{\omega_0}^{\nu}\int_{\nu_1-\nu-\omega_0}^{-\omega_0}] \; d\nu_1 \; d\nu_2 \; (Integrand) \end{aligned} \tag{7}$$

with the integrand being

$$\begin{aligned} S(z,\nu_1)S(z,\nu_2)S(z,\nu-\nu_1+\nu_2) \;\; &\chi(-\nu-\omega_0;\nu_1+\omega_0,-\nu_2-\omega_0,\nu-\nu_1+\nu_2+\omega_0) \\ &\int_{R^2} dr \;\; \psi(r;\omega_0+\nu)\psi(r;\omega_0+\nu_1)\psi(r;\omega_0+\nu_2)\psi(r;\omega_0+\nu-\nu_1+\nu_2) \end{aligned}$$

Assumption 5: The spectrum of the pulse is very narrow compared with the frequency variation of ψ, so Laplace asymptotic analysis can be used to simplify the integral above, by removing dependence on fields ψ.

Assumption 6: The spectrum varies slowly with distance, so the term in S_{zz} is ignored.

Assumption 7: The linear wavenumber $\beta(\omega)$ from eq (3) is available as a Taylor series expansion about ω_0. With such a series, the coefficient of S is available as a series in ν.

Form of χ

Assumption 5 also ensures that the susceptibility χ can be approximated by the form

$$\chi = \chi_0\left[\eta + \frac{1}{2}(1-\eta)(L(\frac{\nu_1-\nu_2}{\omega_r}) + L(\frac{\nu-\nu_1}{\omega_r}))\right]$$

with η, χ_0, and ω_r material constants. Frequency ω_r indicates a material absorbtion close to ω_0. $\Re(L)$ and $\Im(L)$ are related by a Hilbert transform (causality), and $\Im(L)$ gives the Raman gain of the dielectric. An integro-differential equation follows to describe the propagation of the pulse.

Taking an inverse Fourier transform of eq (7) gives a partial differential equation to describe the pulse's propagation. By retaining terms to different order in ν and by making assumptions about the form of $\Im(L)$ different effects are included in the equation. At the simplest level, retain only terms of dominant order in ν:

$$iu_z - \frac{1}{2}\beta_0'' u_{\xi\xi} + \eta M|u|^2 u + (1-\eta)Mu \int_{-\infty}^{\xi} d\xi_1 \; b(\xi - \xi_1)|u|^2(\xi_1) = 0,$$

where M is a constant and b is the Fourier transform of $\Im(L)$. The cubic Schrödinger equation follows from assuming $\Im(L) \equiv 0 \equiv b$, i.e. that there is no Raman gain.

Coupling of pulses

To increase complexity to more than a single pulse in a coupled system, the modelling is immediate.

Assumption 8: The dispersion of the coupling mechanism is small, i.e. the strength of the coupling between the two states is constant over the range of wavelengths for which the pulses' spectra are nontrivial. Given this, the coupling coefficients of the cw systems considered above are inserted directly to couple the nonlinear Schrödinger equations describing pulse propagation in each system in isolation.

Analysis

Coupled pulse equations are not amenable to fully analytical solution in general. Numerical methods, and especially the split-step Fourier (beam propagation) method, prevail.

Semi-analytical approximations are also popular. Most notably, a physically realistic assumption is made about the approximate form of the pulse, and then a variational method is applied to get a set of approximate equations.

Invariant Behaviour

Qualitative description of switching in the coupled system is found from consideration of its steady-state behaviour, i.e. when there is no amplitude variations with z. Determination of this is an interesting study in its own right. For cw operation, eq (5) gives coupled algebraic equations for the normal modes. For pulses, the pertinant equations give coupled ODEs for the true solitons of the coupled nonlinear optical system.