Modulational Instability in Discrete Systems

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ABSTRACT

We present two systems where the dynamics of modulated waves can be modeled by a generalized discrete nonlinear Schrödinger equation.
In the first one, we consider the influence of discreteness in a real nonlinear electrical lattice [1]. Theoretically, we show that the system of nonlinear equations governing the physics of the electrical network can be approximated by a generalized discrete equation which interpolates between the Ablowitz-Ladik and Discrete Self-Trapping equations [2,3]:

$$j\frac{d\Psi_n}{dt} + [\Psi_{n+1} + \Psi_{n-1}] + [\mu(\Psi_{n+1} + \Psi_{n-1}) + 2\nu\Psi_n]\,|\Psi_n|^2 = 0 \ , \qquad (1)$$

with coefficients

$$\begin{cases} \mu = \dfrac{1}{A^2} \\ \nu = -\dfrac{2\omega^2 + \omega_0^2 + 2u_0^2}{2u_0^2 A^2}. \end{cases}$$

This so-called IN-DNLS equation admits staggered localized modes solutions, i.e. localized modes with the neighboring sites oscillating out of phase. Note however that for the real lattice, neither μ nor ν can be zero because their values are determined by those of the components.

¿From this analysis, the possibilities for the system to present modulational instability (MI) have been investigated: we find theoretically that MI only occurs in the wavenumber range $\pi/2 < k < \pi$. Experimentally, we do observe MI in this range, except in the vicinity of the zero-dispersion point, where the IN-DNLS model fails. Moreover, we have the following results:

- we observe MI for frequencies above the linear cutoff frequency f_{max} of the system;
- we observe staggered localized modes ($k = \pi$) for which the whole frequency spectrum is concentrated above f_{max}. These modes undergo strong velocity fluctuations when they propagate, due to lattice dispersion.

Next, in order to study such localized wave packets in molecular lattices, we consider a one-dimensional molecular model with a few number of atoms [4]. Each atom is submitted to an harmonic on-site potential with characteristic frequency ω_0 and is anharmonically coupled to its nearest neighbors, accordingly to the hamiltonian:

$$H = \sum \frac{m\omega_0^2}{2} u_n^2 + \sum \frac{m}{2} \left[\frac{du_n}{dt}\right]^2 + \sum V(u_{n+1} - u_n) \tag{2}$$

where $u_n(t)$ denotes the displacement of atom n. Then, the dynamics of the lattice can be described by an IN-DNLS equation similar to eq.(1). As for the first system, MI occurs when the wavenumber exceeds $\pi/2$, and localized staggered modes ($k = \pi$) are theoretically expected. These predictions are then numerically checked on a chain of six atoms with periodic boundary conditions. We find that 60% of the total energy of the chain localise, with a recurrence period, successively on each atom.

References

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