Numerical Analysis of Differential Equations

Conference to mark the retirement of Professor David Sloan

September 9-10th, 2004

University of Strathclyde

Incorporating the Thirteenth Scottish Computational Mathematics Symposium

Thursday 9 September 2004

9:00 - 9:20	Tea/Coffee on arrival
9:20 - 9:25	Welcome
9:25 - 10:10	Nick Trefethen (Oxford University)
10:15 - 11:00	Bill Morton (University of Bath)
11:00 - 11:30	Tea/Coffee
11:30 - 12:15	David Griffiths (University of Dundee)
12:20 - 1:05	Bengt Fornberg (University of Colorado)
1:05 - 2:00	Lunch
2:00 - 2:45	Andrew Stuart (University of Warwick)
2:50 - 3:35	Alastair Spence (University of Bath)
3:35 - 4:05	Tea/coffee
4:05 - 4:50	Bob Russell (Simon Fraser University)
4:55 - 5:40	Bill Sloan (University of Glasgow)
7:30	Pre-dinner drinks at the Lord Todd
8:00	Dinner at the Lord Todd

Friday 10 September 2004

9:25 - 10:10	Weizhang Huang (University of Kansas)
10:15 - 11:00	Endre Süli (Oxford University)
11:00 - 11:30	Tea/Coffee
11:30 - 12:15	Arieh Iserles (University of Cambridge)
12:20 - 1:05	Mark Ainsworth (University of Strathclyde)

Abstracts in alphabetical order

Mark Ainsworth

Dispersive and Dissipative Behaviour of High Order Galerkin Schemes

We consider the dispersive properties of Galerkin finite element methods for wave propagation. The dispersive properties of conforming finite element schemes are analysed in the setting of the Helmholtz equation and an explicit form of the discrete dispersion relation is obtained for elements of arbitrary order. It is shown that the numerical dispersion displays three different types of behaviour depending on the size of the order of the method relative to the mesh-size and the wave number. Quantitative estimates are obtained for the behaviour and rates of decay of the dispersion error in the differing regimes.

We then turn our attention to the dispersive properties of a high order discontinuous Galerkin scheme. In the small wave-number limit, we show the discontinuous Galerkin gives a higher order of accuracy than standard Galerkin. If the mesh-size is fixed and the order is increased, it is shown that the dissipation and dispersion errors decay at a super-exponential rate when the order is sufficiently large, and we give sharp bounds on where the transition occurs. Finally, we analyse the case of most practical interest where the order is chosen on the envelope where the super-exponential convergence sets in using sharp uniform asymptotics.

David Griffiths

Advection through irregular grids

(joint with M Babatin, University of Dundee)

There is surprisingly little information in the literature regarding the behaviour of numerical solutions of advection equations through irregular grids. Using one of the simplest semi-discrete models we review and rework the ideas of:

- Kreiss & Oliger (1973), on the transition between two uniform grids, one fine and the other coarse,
- Giles & Thompkins (1985), on motion through slowly varying grids using techniques developed for dispersive PDEs,
- Vichnevetsky (1989), where an analogy with Schrödinger's equation from Quantum Mechanics is used to describe waves trapped in regions of highest grid refinement.

We conclude by analysing a phenomenon described in Gresho & Sani (1998) whereby the solutions of certain numerical methods on perturbations of uniform grids develop high frequency oscillations whereas others do not. We show how these oscillations can be understood through dispersion relations (eigenvalue problems) and the interactions of Fourier modes in the initial data with those in the grid perturbations.

Bengt Fornberg

Pseudospectral methods for high order initial-boundary value problems

When pseudospectral approximations are used for space derivatives, one often encounters spurious eigenvalues. These can lead to severe time stepping difficulties for PDEs. This is especially the case for equations with high order derivatives in space, requiring multiple conditions at one or both boundaries. Following up on some pioneering contributions to this issue by Dave Sloan and his collaborators, we note here that a very simple-to-implement fictitious point approach circumvents most of these difficulties. The new approach is tested on the Kuramoto-Savishinsky equation. We conclude by noting that highly accurate solution methods for initial-boundary value problems makes time-space corner singularities for such problems an issue of increasing importance.

Weizhang Huang

The Equidistribution Principle for Multi-Dimensional Mesh Adaptation

The equidistribution principle plays a fundamental role in mesh adaptation. In an early work (”A simple adaptive grid method in two dimensions”, SIAM J. Sci. Comput. 15 (1994), 776-797), Dave Sloan and I described an understanding of the principle and found that it cannot be satisfied exactly in general on a structured mesh. An algorithm satisfying the equdistribution principle in a local manner was further proposed for mesh adaptation in two dimensions. In this talk, I shall describe some new understandings of the principle and present a variational mesh adaptation strategy that approximately realizes equidistribution in multi-dimensions.

Arieh Iserles

Highly oscillatory quadrature and its applications

(joint with Syvert Nørsett, Trondheim University)

In this talk I will review recent advances in understanding and implementing methods for quadrature with highly oscillatory kernels. We develop two methods, one based on an asymptotic expansion and the other on interpolation, that afford very precise approximation in the presence of high oscillation and critical points, in one or more dimensions. Time allowing, I will describe some of the applications of these methods to Fredholm equations of the second kind and ordinary and partial differential equations with rapidly oscillating solutions.

Bill Morton

Approximating hyperbolic equations

The numerical approximation of partial differential equations is all about reproducing their key properties. This has a very long history, going back to the exploitation of maximum principles, variational principles, characteristic structures, etc.; the latest addition to the list is variously called 'geometric' or 'symplectic' integration. We will describe three examples of approximating hyperbolic pdes which illustrate the effectiveness of these ideas.

Bob Russell

Some Moving Mesh Methods for Solving Time-Dependent PDEs

In the context of solving time-dependent parabolic PDEs, some basic moving mesh procedures will be reviewed. We consider two basic types, both basically involving the computation of a coordinate transformation from physical coordinates to computational coordinates. One minimizes a suitable variational form for this coordinate transformation, while the other computes the mesh velocities directly. Relations between these methods are discussed, as well as the challenges facing each when solving higher dimensional PDEs. We end by discussing interesting relations between the moving mesh problem and a variety of general problems in science and engineering.

Bill Sloan

Mathematical modelling of engineered biological systems: the challenge of improving public health to save lives

Research into the numerical solution of partial differential equations during the latter half of the 20th century has been driven by problems in the physical sciences. During this period biology has undergone a radical transformation from qualitative to quantitative science, which has been brought about by a revolution in the characterisation of an organism's DNA molecules. Biological systems are now measurable and biologists are beginning to speculate on the existence of fundamental rules underlying the patterns they observe. This makes the field ripe for the intervention of mathematicians. Here, it is argued that the rewards for applied mathematicians willing to bring their skills to bear on biological problems could be immense. Not only because of the mathematical challenges these problems offer but also because solving them may have a direct and immediate impact on the health and wealth of individuals and populations. The research described here is an example of one such challenge and application. A model that appears to describe microbial community assembly is developed, which takes the form of a multi-dimensional Fokker-Planck equation. Solutions of this will inform the design of affordable small-scale waste treatment facilities in developing countries that could ultimately save lives.

Alastair Spence

Inexact Inverse Iteration for Nonsymmetric Eigenvalue problems

Large scale nonsymmetric eigenvalue problems arise in a variety of physical situations; for example, in the analysis of stability of fluid flows, or in clustering techniques for networks. Typically, only a few eigenvalues are required and so techniques like Arnoldi's method are applied, often to a "shift-invert" form of a matrix with linear solves carried out iteratively. In this talk we will study Inexact Inverse Iteration, that is, inverse iteration where the linear solves are implemented inexactly by some iterative technique. This is the simplest inexact iterative method, but a good understanding of this basic approach provides a foundation from which more sophisticated inexact techniques can be understood.

We shall first discuss the convergence theory for Inexact Inverse Iteration and a reformulation called "Inverse Correction". Next we discuss the effects of implementing the linear solves using preconditioned GMRES. Numerical results will be presented to illustrate various points in the theory.

Andrew Stuart

Homogenization for Inertial Particles

(joint with Jochen Voss and Petter Wiberg, University of Warwick)

We describe a stochastic PDE based approach to sampling paths of SDEs, conditional on observations. The SPDEs are derived by discretizing the problem, then applying the Langevin MCMC method, and passing to the limit of infinite dimensions. Various applications are described including sampling paths subject to two end point conditions (bridges) and nonlinear filter/smoothers.

Endre Süli

Discontinuous Galerkin Methods in Computational Continuum Mechanics: Stability, Accuracy, Adaptivity

Discontinuous Galerkin Finite Element Methods (DGFEMs) were introduced in the early 1970s for the numerical solution of first–order hyperbolic problems. Simultaneously, but quite independently, they were proposed as nonstandard schemes for the approximation of linear and semilinear second–order elliptic equations. The recent upsurge of interest in this class of techniques has been stimulated by the computational convenience of DGFEMs due to a high degree of locality, the need to approximate advection–dominated diffusion problems without excessive numerical stabilisation, the necessity to accommodate high–order *hp*– and spectral element discretisation for first–order hyperbolic equations and advection–diffusion problems, and the desire to handle nonlinear hyperbolic problems in a locally conservative manner and without auxiliary numerical stabilisation.

The lecture will review recent developments in the field concerning the *a priori* and *a posteriori* error analysis of these methods for degenerate elliptic partial differential equations and PDEs of mixed elliptic-hyperbolic-parabolic type. We shall touch on the question of *hp*-adaptivity, illustrating the theoretical ideas by examples that arise from computational electro-chemistry and compressible gas dynamics. Finally, we shall survey some new analytical results concerning the accuracy of these methods for quasilinear elliptic equations that arise as mathematical models of steady incompressible non-Newtonian flow.

Nick Trefethen

Interpolation in Chebyshev Points

One of the areas to which Dave Sloan has made notable contributions is the solution of differential equations by spectral collocation, a method based on polynomial interpolation in Chebyshev points. Such interpolants can be computed fast and accurately, even if there are tens of thousands of interpolation points. However, the convergence is so fast for smooth functions that usually a few dozen points are enough. This talk will review some of the fundamental theorems that justify this powerful technique.