

SCMS 02

SCOTTISH COMPUTATIONAL MATHEMATICS SYMPOSIUM 2002

10.00 am – 4.30 pm on Friday 20 September 2002

Court Senate Lecture Theatre
University of Strathclyde
Glasgow

The meeting is supported by the **London Mathematical Society**

Programme

- 10.00 Welcome
- 10.05 Professor Mike Christie (Petroleum Engineering, Heriot-Watt)
Uncertainty quantification in oil recovery predictions
- 10.55 Coffee/Tea
- 11.25 Professor Nancy Nichols (Mathematics, Reading University)
Data assimilation in large scale dynamical models
- 12.15 Lunch
- 13.30 Professor John Barrett (Mathematics, Imperial College)
Finite element approximation of a void electromigration model
- 14.20 Professor Nigel Weatherill (Civil Engineering, UCW, Swansea)
Issues of mesh quality in computational engineering
- 15.10 Coffee/Tea
- 15.40 Professor Will Light (Mathematics, Leicester University)
The modern context and technology for approximation with radial basic functions
- 16.30 End

TALKS

Mike Christie

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Uncertainty quantification in oil recovery predictions

Predicting the performance of oil reservoirs is inherently uncertain: data constraining the rock and rock-fluid properties is available at only a small number of spatial locations, and other measurements are integrated responses providing limited constraints on model properties. Calibrating a reservoir model to observed data is time consuming, and it is rare for multiple models to be 'history matched'. Uncertainty quantification usually consists of identifying high-side and low-side adjustments to the base case.

This talk will describe a technique for quantifying uncertainty in reservoir performance prediction. The method, known as the Neighbourhood Algorithm, is a stochastic sampling algorithm developed for earthquake seismology. It works by adaptively sampling in parameter space to bias the sampling to regions of good fit to data. The algorithm evaluates the high dimensional integrals needed for quantifying the posterior probability distribution.

We demonstrate the performance of the algorithm on a synthetic case originally developed for use in the SPE Comparative Solution Project. Reservoir oil and water rates, and average reservoir pressure are computed from the fine grid solution and the reservoir performance data for the first 300 days is used as input. We generated multiple coarse grid reservoir models and assessed the misfit in oil rate and pressure. We then use the Neighbourhood Algorithm to generate multiple models that match observed history data and predict the range of possible reservoir rates out to 2000 days. The results presented will focus on the ability of the method to sample effectively from the posterior distribution.

Nancy Nichols

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Data assimilation in large scale dynamical models

Mathematical models for simulating physical, biological and economic systems are now often more accurate than the data that is available to drive them. In particular, complete information describing the initial state of an evolutionary system is seldom known. The technique of data assimilation aims to produce accurate estimates of all the initial (and future) state variables of the system by incorporating measured observations of the output from the system into the dynamical model over an interval of time. The optimal estimates minimize a variational principle where the model equations are treated as strong

constraints. A variety of data assimilation schemes for treating very large nonlinear systems will be introduced here and the application of these schemes will be illustrated with simple test examples.

In reality, the model does not represent the system behaviour exactly and errors arise due to lack of resolution and inaccuracies in physical parameters, boundary conditions and forcing terms. A technique for estimating systematic and time-correlated errors as part of the assimilation procedure will also be described here. The modified method determines a correction term that compensates for model error and leads to improved predictions of the system states. The effectiveness of the new procedure is demonstrated in several applications.

John Barrett

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Finite element approximation of a void electromigration model

We consider a finite element approximation of a nonlinear degenerate parabolic system of Cahn-Hilliard type. In the limit of the interfacial parameter going to zero, this “phase field” system models the evolution of voids by surface diffusion in an electrically conducting solid. In addition to showing stability bounds for our approximation, we prove convergence, and hence existence of a solution to this nonlinear degenerate parabolic system in two space dimensions. Furthermore, an iterative scheme for solving the resulting nonlinear discrete system is introduced and analysed. Finally, some numerical experiments are presented.

Nigel Weatherill

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Issues of mesh quality in computational engineering

Fundamental to finite element and finite difference analysis is a spatial discretisation of the domain i.e. the construction of a mesh. For simple geometries, this is not an issue. However, when the geometry is complicated, the construction of an appropriate mesh can be a major challenge. In fact, the automatic construction of a mesh is often cited as *the* major problem preventing the more widespread use of computational analysis via finite element and finite difference analysis.

The presentation will begin with a review of the different approaches to mesh generation. The advantages and limitations of the different approaches will be highlighted using examples from real-world problems in science, engineering and medicine.

Although the state-of-the-art in mesh generation has advanced in the last 10 years, there are still major issues that need to be addressed. Using examples from computational fluid dynamics and computational electromagnetics, the presentation will highlight some new challenges and opportunities for significant advances. Some recent developments using new and innovative ideas will be presented.

Will Light

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The modern context and technology for approximation with radial basic functions

Let $\psi : \mathbf{R}^n \rightarrow \mathbf{R}$ be a given function. Our talk will concentrate on interpolating data at m points x_1, \dots, x_m by m translates of this basic function ψ . These translates will always involve the interpolation points themselves, so that the interpolant has the form

$$s(x) = \sum_{i=1}^m a_i \psi(x - x_i), \quad x \in \mathbf{R}^n$$

The most common form of ψ is a radial basic function so that $\psi(x) = \phi(|x|)$, where $|\cdot|$ is the usual Euclidean norm in \mathbf{R}^n . Given data d_1, \dots, d_m in \mathbf{R} , we then solve the equations

$$d_j = s(x_j) = \sum_{i=1}^m a_i \psi(x_j - x_i), \quad j = 1, \dots, m.$$

Why is this scheme so beloved by people handling multivariate data? Well, the folklore has it that

1. the interpolant exists independent of the location of the data points (except that the data points must be distinct), and so can handle scattered data
2. some special choices of ϕ lead to interpolants with very nice properties
3. very good asymptotic error estimates are available. These error estimates are indicative of the performance of the error in a typical application
4. the method is simple to programme, and can handle data in high dimensions with no difficulty at all.

Of course, these statements are precisely what I claimed them to be – folklore. The practical outflow from some of them is very different. I will discuss in broad terms the significant developments which have taken place over the last 10 years or so, which have culminated recently in translating the folklore into reality. Part of this progress has been underwritten by some very clever mathematics, and part of it by advances in computing technology. I hope to answer the following questions:

1. why do the equations have a unique solution, and where does the basic idea for this method originate?
2. what are some of the useful indications that the error performance of these methods is so good, and how are they couched mathematically?
3. how does one go about inverting the interpolation matrices that result, particularly if one has a very large number of data sites?

In my talk I will aim chiefly to appeal to the non-expert, although I hope there will also be a number of items of interest to the experts.