Aims

Abstract algebra is the study of algebraic structures. The course aims to provide an introduction to some of the most fundamental algebraic structures encountered in algebra and geometry: groups, rings, and fields.

Syllabus

Groups of transformations: Group of symmetries of an equilateral triangle. Transformation groups. First look at the symmetric group: Matrix and cycle decomposition for permutations. (4 lectures)

Axiomatic approach to groups: Binary operations. Axioms for a group and consequences. Cayley tables. Groups coming from sets of numbers. Groups coming from modular arithmetic. (5 lectures)

Subgroups: The subgroup test. Order of elements. Subgroups generated by a family of elements. The symmetric group and the alternating group. Lagrange’s Theorem. (5 lectures)

Homomorphisms: Homomorphisms. Kernels and images. Isomorphisms and group invariants. Cayley’s Theorem. (5 lectures)

Quotient groups: Normal subgroups. Quotient groups. The first isomorphism theorem. (3 lectures)

Axiomatic approach to rings and fields: Axioms for rings and fields. Examples. Subrings and ideals. Homomorphisms and quotient rings. (4 lectures)


Sums of squares: Squares in \( \mathbb{Z}/p\mathbb{Z} \). Integers that can be written as a sum of two squares. (2 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: up to 15% by class tests or other continuous assessment at least 85% by end of course 2-hour exam
Resit Type: exam
By the end of the course, students should be able to:

- perform standard group computations with linear isometries of the plane and permutations of a finite set
- give standard examples of groups, rings and fields
- carry out simple deductions using axioms for binary operations, groups, rings, and fields
- construct and complete Cayley tables for small groups
- understand the notion of subgroup and determine whether a given subset is a subgroup
- apply Lagrange’s theorem to study the subgroups of a given group
- understand the notions of homomorphism and isomorphism in groups, rings, and fields
- understand the notion of normal subgroup and determine whether a given subgroup is normal
- understand the notion of ideal and determine whether a given subset of a ring is an ideal
- understand the construction of a quotient in groups and rings
- apply the first isomorphism theorem to prove that two groups/rings/fields are isomorphic
- understand polynomial rings and their use to construct finite fields
- understand the significance of the properties of rings in solving certain diophantine equations