Aims

To give an introduction to numerical linear algebra. To study techniques and analyze algorithms for solving linear systems of equations and eigenvalue problems. By the end of the course, students should be able to apply the methods and algorithms listed below, and carry out the associated analysis of errors, convergence and operation counts.

Syllabus

**Basic linear algebra:** Basic linear algebra: Matrices, FLOPS, matrix product, Gaussian elimination, matrix inverse, block matrices, sparse matrices, discrete Laplacian, eigenvalues, functional calculus and the ordinary differential equation $y' = Ay$. *(7 lectures)*

**Matrix decompositions:** Matrix decompositions: QR, LU, LL* and LDL* decomposition. Operation counts. Least squares problems. *(12 lectures)*

**Iterative algorithms for the solution of linear systems:** Iterative algorithms for the solution of linear systems: Jacobi, Gauss-Seidel, GMRES. Convergence analysis. *(9 lectures)*

**Iterative algorithms for eigenvalue problems:** Power, shifted inverse power methods. Convergence analysis. Francis algorithm. *(5 lectures)*

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** 10% by class tests or other continuous assessment
90% by end of course 2-hour exam

**Resit Type:** exam
By the end of the course, students should be able to:

- write Matlab procedures for the methods considered in the course.
- understand linear systems of equations, vectors, matrices.
- understand how to count FLOPS in algorithms.
- understand block, sparse, lower and upper triangular matrices and memory requirements.
- understand how to multiply matrices.
- understand Gaussian elimination and estimate FLOPS.
- understand the need for pivoting for reducing round-off errors in GE algorithms.
- understand how to compute the matrix inverse by the companion matrix approach and estimate FLOPS.
- understand the sparse 1d and 2d Laplacian matrices.
- understand the $LU$ decomposition and its relationship with GE and estimate FLOPS.
- understand how to compute the determinant from the $LU$ decomposition.
- understand the $L DL^*$ decomposition for symmetric matrices and how to compute it from GE.
- understand the $LL^*$ decomposition for symmetric and positive definite matrices and how to compute it from the $L DL^*$ decomposition.
- understand orthonormal basis, columnwise orthonormal matrices, the Gram-Schmidt procedure and estimate FLOPS.
- understand how to compute the $QR$ decomposition incrementally from the Gram-Schmidt procedure and estimate FLOPS.
- understand how to solve overdetermined problems in the least-squares sense using the $QR$ decomposition.
- understand the general ideas behind iterative algorithms for the solution of linear systems.
- know the Jacobi, Gauss-Seidel and Successive Over-Relaxation methods and analysis in terms of the iteration matrix.
- understand eigenvalues and eigenvectors.
- understand the GMRES algorithm, be able to compute GMRES iterates and analyze the convergence and the FLOPS count.
- understand the power method, the shifted inverse power method and be able to analyze the convergence of these algorithms and the FLOPS count.
- understand deflation for finding the eigenvalues of block upper-triangular matrices.
- understand how to compute the conjugate $Q^{-1}AQ$ of a matrix and use it to find eigenvalues.
- understand reduction to upper Hessenberg form using Householder reflections.
- understand deflation for reducible upper Hessenberg matrices.
- understand the Francis $QR$ iteration for calculating eigenvalues, including Householder reflections.