Aims

The objective of the module is to introduce and develop the methods of vector analysis. These methods provide a natural aid to the understanding of geometry and some physical concepts. They are also a fundamental tool in many theories of Applied Mathematics.

Syllabus

**Vector Algebra and Geometry:** Vector addition and scalar multiplication. Scalar and vector products. Equations of lines and planes. Curves and surfaces; parametric and non-parametric equations of curves and surfaces. (5 lectures)

**Vector Differentiation:** Differentiation of vector valued functions with respect to a scalar. Geometry of curves. Scalar and vector fields. Gradient of a scalar field, and divergence and curl of a vector field. Sum and product rules for these differentiation operators. Second order vector operators. Directional derivatives. Normal and tangent plane to a surface. Solenoidal and irrotational fields. (8 lectures)

**Vector Integration:** Curvilinear line integrals. Surface integrals. The divergence theorem, Green's theorem and Stoke's theorem. (10 lectures)

**Curvilinear Coordinate Systems:** Coordinate free vector derivatives. Vector derivatives in curvilinear coordinates. Spherical, polar and cylindrical coordinates (4 lectures)

**Potential Theory:** Gradient fields. Rotation fields. Harmonic functions. Helmholtz's fundamental theorem of vector calculus. (6 lectures)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** up to 15% by class tests or other continuous assessment

at least 85% by end of module 2-hour exam

**Resit Type:** examination
By the end of the course, students should be able to:

- Calculate scalar and vector products.
- Find the vector equations of lines and planes.
- Understand the parametric equations of curves and surfaces.
- Differentiate vector functions of a single variable.
- Calculate velocity and acceleration vectors for moving particles.
- Understand and be able to find the unit tangent vector, the unit principal normal and the curvature of a space curve.
- Find the gradient of a function.
- Find the divergence and curl of a vector field and prove identities involving these.
- Use the gradient operator to calculate the directional derivative of a function.
- Calculate the unit normal at a point on a surface.
- Recognise irrotational and solenoidal vector fields.
- Evaluate line and surface integrals.
- Understand the various integral theorems relating line, surface and volume integrals.