Aims

The course aims to give an understanding of linear and nonlinear ordinary differential equations and systems of equations and to show how ordinary differential equations are important in mathematical modelling.

Syllabus

**Introduction to differential equations:** Revision of first-order equations, exact equations, existence and uniqueness of solutions, direction fields, exactly solvable second-order equations. (4 lectures)

**Linear systems of ODEs:** Fundamental sets of solutions, equations with constant coefficients, Wronskians, inhomogeneous equations, variations of parameters, solution of linear systems by matrix methods. (9 lectures)

**Laplace transforms:** Calculation of transforms, solution of linear equations and systems, inverse transforms, equations with discontinuous or impulsive forcing terms, convolutions. (4 lectures)

**Boundary value problems:** Existence and uniqueness of solutions, Green’s functions (4 lectures)

**Sturm Liouville problems:** Eigenvalues and eigenfunctions, orthogonality of eigenfunctions, eigenfunction expansions. (4 lectures)

**Phase Planes:** Equilibrium points, phase planes for nonlinear second-order equations, the pendulum equation, phase planes for linear systems, classification of equilibrium points, stability. (7 lectures)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** up to 15% by class tests or other continuous assessment

at least 85% by end of module 2-hour exam

**Resit Type:** exam
By the end of the course, students should be able to:

- recognize and solve first-order ODE's which are separable, linear, can be solved by changing variable (e.g. homogeneous, Bernouilli type), exact.
- understand how Picard's theorem can be applied to prove the the existence and uniqueness of solutions of initial value problems.
- sketch the graphs of solutions of first-order equations by first drawing appropriate direction fields.
- use MAPLE to find direction fields and to plot solutions of equations.
- find fundamental sets of solutions of homogeneous second-order equations with constant coefficients, of Euler type, and where one solution can be found by inspection.
- understand Wronskians; be able to state and prove Abel's theorem and verify that the theorem holds for given equations.
- understand the connection between the solutions of homogeneous and inhomogeneous equations and solve inhomogeneous equations using the methods of undetermined coefficients and variation of parameters.
- find the general solution of linear systems of two ODE's with constant coefficients in terms of the eigenvalues and eigenfunctions of an appropriate matrix.
- calculate Laplace transforms and know the basic properties of the transforms.
- use Laplace transforms to solve initial value problems for second-order ODE's.
- use Laplace transforms to solve problems involving discontinuous functions and Dirac delta functions, including problems arising in oscillating springs.
- understand convolutions and be able to solve integral equations involving convolutions by using Laplace transforms.
- invert Laplace transforms by using the convolution theorem.
- use Laplace transforms to solve linear systems of ODE's.
- understand the difference between initial value problems and boundary value problems.
- show that sometimes boundary value problems have no solution.
- be able to solve boundary value problems under appropriate conditions when no Green's function exists.
- find the eigenvalues and eigenfunctions of boundary value problems.
- understand the completeness and orthogonality of families of eigenfunctions.
- find general eigenfunction expansions.
- know what is meant by a phase plane.
- sketch the phase plane of an autonomous second-order nonlinear ODE by first finding the trajectories.
- draw the phase plane corresponding to the nonlinear pendulum equation.
- classify equilibrium points for systems of two linear ODE's with constant coefficients and sketch the corresponding phase planes.
- understand the concept of stability.