Aims

The objective of the module is to introduce some fundamental ideas and techniques in Applied Mathematics.

Syllabus

Fourier Analysis: Full and half range Fourier series. (3 lectures)

An introduction to PDEs: Simple PDEs; Separation of Variables; Solution of Laplace’s equation and the wave equation making use of Fourier series. (8 lectures)

Calculus of variations: variational derivative; Euler–Lagrange equations; examples including the Brachistochrone, isoperimetrical, and soap bubble problems; extensions to higher derivatives, several dependent and independent variables; constraints and Lagrange multipliers. (6 lectures)

Lagrangian mechanics: action; Hamilton’s Principle; Lagrange’s equations; examples including the Kepler and simple pendulum problems; derive incompressible Euler equations; Hamilton’s equations; Poisson brackets and the Hamilton–Jacobi partial differential equation, Noether’s theorem. (14 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 15% by class tests or other continuous assessment
85% by end of course 2-hour exam
Resit Type: exam
By the end of the course, students should be able to:

- find the Fourier sine and cosine series of simple functions on $[-L, L]$
- find the half-range Fourier sine and cosine series of functions on $[0, L]$
- understand the concept of a PDE
- understand the meaning and application of Laplace's equation and the wave equation
- understand and be able to use the separation of variables approach
- solve Laplace's equation in 2D with various boundary conditions using separation of variables and Fourier analysis
- solve the wave equation with various initial conditions using separation of variables and Fourier analysis
- derive the Euler–Lagrange equations for the extremizer of a functional;
- solve the Euler–Lagrange equations for simple examples;
- perform both of the previous two exercises for functionals involving higher derivatives and/or more than one dependent and/or independent variables;
- use Lagrange multipliers to solve problems with constraint;
- define action and state Hamilton's Principle;
- derive Lagrange's equations of motion and use them to solve for the dynamics of simple examples, eg. Kepler and simple pendulum problems;
- derive Hamilton's equations;
- understand the relevance and use of the Hamilton–Jacobi partial differential equation;
- understand how Euler's equations for an incompressible ideal fluid can be derived from Hamilton's Principle;
- exploit symmetries to solve simple mechanics problems
- understand the relation between symmetries and conservation laws

Content: September 2017