Aims

The objective of the module is to introduce some fundamental ideas and techniques in Applied Mathematics.

Syllabus

**Fourier Analysis:** Full and half range Fourier series.  
*(4 lectures)*

**An introduction to PDEs:** Simple PDEs; Separation of Variables; Solution of the heat equation, Laplace’s equation and the wave equation making use of Fourier series.  
*(8 lectures)*

**Calculus of variations:** variational derivative; Euler–Lagrange equations; examples including the Brachistochrone, isoperimetrical, and soap bubble problems; extensions to higher derivatives, several dependent and independent variables; constraints and Lagrange multipliers.  
*(6 lectures)*

**Lagrangian mechanics:** action; Hamilton’s Principle; Lagrange’s equations; examples including the Kepler and simple pendulum problems; Poisson brackets; Noether’s theorem.  
*(12 lectures)*

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** 15% by class tests or other continuous assessment
85% by end of course 2-hour exam

**Resit Type:** exam
By the end of the course, students should be able to:

- find the Fourier sine and cosine series of simple functions on $[-L, L]$
- find the half-range Fourier sine and cosine series of functions on $[0, L]$
- understand the concept of a PDE
- understand the meaning and application of the heat, Laplace’s and wave equations
- understand and be able to use the separation of variables approach
- solve the heat equation in 1D with various boundary conditions using separation of variables and Fourier analysis
- solve Laplace’s equation in 2D with various boundary conditions using separation of variables and Fourier analysis
- solve the wave equation with various initial conditions using separation of variables and Fourier analysis
- derive the Euler–Lagrange equations for the extremizer of a functional
- solve the Euler–Lagrange equations for simple examples
- perform both of the previous two exercises for functionals involving higher derivatives and/or more than one dependent and/or independent variables
- use Lagrange multipliers to solve problems with constraint
- define action and state Hamilton’s Principle
- derive Lagrange’s equations of motion and use them to solve for the dynamics of simple examples, eg. Kepler and simple pendulum problems
- derive Hamilton’s equations
- understand Poisson brackets
- exploit symmetries to solve simple mechanics problems
- understand the relation between symmetries and conservation laws

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