Aims

This module offers an introduction to the ideas of number theory and geometry to students specialising in Mathematics. Technical skills acquired at Level 1 will be applied to develop the ideas of these two vital strands of mathematical thought, and to offer further insights into mathematical reasoning and the art of proof in a concrete setting.

Syllabus


Prime numbers: Factorisation in $\mathbb{Z}$. Primes and their distribution. Fermat and Mersenne primes. (5 lectures)

Pythagorean triples: Classification of Pythagorean triples. (2 lectures)

Congruences: Congruence as an equivalence relation. Solution of linear and simultaneous congruences. (4 lectures)

Multiplicative functions: Summing over divisors, Euler’s totient function, Fermat and Euler theorems, applications to cryptography. (5 lectures)

Irrational numbers: Irrational, algebraic and transcendental numbers. Countable sets. Diophantine approximation. (2 lectures)

Transformation geometry: Classification of plane isometries and their elementary properties. (11 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 15% by class tests or other continuous assessment
85% by end of course 2-hour exam
Resit Type: exam only
By the end of the course, students should be able to:

- understand the arithmetic and order properties of $\mathbb{Z}$
- understand the properties of the divisibility relation on $\mathbb{Z}$
- carry out the Euclidean algorithm to find greatest common divisors
- find Pythagorean triples and understand their classification
- understand the Fundamental Theorem of Arithmetic
- find prime numbers using a sieve method
- prove that there are infinitely many primes
- understand qualitative facts about the distribution of primes
- find primes in certain families
- understand congruence as an equivalence relation on $\mathbb{Z}$
- solve linear and simultaneous congruences
- understand the properties of Euler's $\phi$—function
- understand the notion of a plane isometry or motion and their basic properties
- work with the matrix representation of plane isometries
- classify a given plane isometry