Aims

This is a second-level module in Linear Algebra aimed at students specializing in Mathematics, Statistics, or Actuarial Mathematics. The course aims to provide sufficient knowledge of matrix theory and of the solution of systems of linear equations for use in later courses in mathematics and statistics; to give an understanding of the basic concepts of linear algebra; and to develop the ability to solve problems and prove theorems involving these concepts.

Syllabus

**Euclidean space:** Vector spaces $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^n$, Matrices, Basic matrix operations, Determinants. *(4 lectures)*

**Systems of linear equations:** Gaussian elimination, Results on homogeneous and inhomogeneous systems, Matrix inversion. *(4 lectures)*

**Vector spaces:** Definition and examples of vector spaces, Subspaces, Span, Linear independence, Bases and dimension. *(5 lectures)*

**Inner product spaces:** Scalar or Inner products, Cauchy-Schwartz inequality, Orthogonality, Orthogonal projection, Orthonormal bases, Gram-Schmidt process, Vector products. *(5 lectures)*

**Linear transformations:** Row and Column rank of a matrix, Applications to systems of equations, Range, Kernel, Rank and Nullity, Invertibility of linear transformations, Linear transformations and matrices. *(9 lectures)*

**Eigenvalues and eigenvectors:** Calculation of Eigenvalues and Eigenvectors, Symmetric matrices, Diagonalisation of a matrix, Cayley Hamilton theorem, Iterates of matrices, applications to quadratic forms. *(6 lectures)*

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** 30% by class tests or other continuous assessment

70% by end of module 2-hour exam

**Resit Type:** exam
By the end of the course, students should be able to:

- Use the Gaussian elimination procedure to determine whether a given system of simultaneous linear equations is consistent, and if so to find the general solution. Invert a matrix by the Gaussian elimination method.
- Understand the concepts of vector space and subspace, and apply the subspace test to determine whether a given subset of a vector space is a subspace.
- Understand the concepts of linear combination of vectors, linear (in)dependence, spanning set, and basis. Determine if a set of vectors is linearly independent and spans a given vector space.
- Find a basis for a subspace, defined either as the span of a given set of vectors, or as the solution space of a system of homogeneous equations.
- Understand the concept of inner product in general, and calculate the inner products of two given vectors.
- Understand the concept of orthogonal projection and how to explicitly calculate the projection of one vector onto another one. Use the Gram-Schmidt method to convert a given basis for a vector space to an orthonormal basis. Understand (geometrically) the concept of the vector product and being able to calculate the vector product of two given vectors.
- Find the coordinates of a given vector in terms of a given basis - especially in the case of an orthogonal or orthonormal basis.
- Calculate the rank of a given matrix and, from that, the dimension of the solution space of the corresponding system of homogeneous linear equations. Calculate the determinant of $2 \times 2$ and $3 \times 3$ matrices.
- Understand the concepts of linear transformation, range and kernel (nullspace). Understand the concept of invertibility of a linear transformation including injectivity and surjectivity.
- Know and apply the Rank-Nullity Theorem.
- Compute the characteristic polynomial of a square matrix and (in simple cases) factorise to find the eigenvalues.
- Determine whether a given square matrix is diagonalisable, and if so find a diagonalising matrix.
- Apply the Cayley-Hamilton Theorem to compute powers of a given square matrix.