Linear Algebra

Aims

Linear Algebra is, at its core, the study of systems of linear equations. Because systems of linear equations are relatively easy to solve, many scientific fields (physics, engineering, economics, etc.) use simplified models where equations are approximated by linear ones, making Linear Algebra a central tool.

The goal of this course is to introduce the basic ideas and techniques of Linear Algebra, and to develop the geometric viewpoint that will allow you to reinterpret and solve many problems using this language. For instance, we will see how techniques from Linear Algebra can be used to approximate statistical data, model dynamics of populations, or even understand the PageRank algorithm used by Google to rank websites.

Syllabus

Systems of linear equations: Gaussian elimination. Results on homogeneous and inhomogeneous systems. (3 lectures)

Vectors: Vectors in $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^n$. Inner product, norm, and angles. Linear combinations of vectors. Bases of $\mathbb{R}^n$. (3 lectures)


Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 15% by class tests or other continuous assessment
85% by end of course 2-hour exam
Resit Type: exam

Content: August 2018
By the end of the course, students should be able to:

- Use Gaussian elimination to determine whether a given system of simultaneous linear equations admits solutions, and if so to find the general solution.
- Understand the concepts of vector space and subspace, and determine whether a given subset of a vector space is a subspace.
- Understand the concepts of linear combination of vectors, linear independence, spanning set, and basis.
- Solve basic problems involving vectors by reinterpreting these problems in terms of systems of linear equations.
- Find a basis for a subspace, defined either as the span of a given set of vectors, or as the solution space of a system of homogeneous equations.
- Understand the concept of linear map, and the various notions associated: nullspace, rank, etc. Calculate the matrix associated to a linear map and perform a change of basis.
- Apply the Rank-Nullity Theorem to determine the dimension of certain subspaces.
- Understand the concept of orthogonal projection and how to explicitly calculate the orthogonal projection of one vector onto a subspace. Use the Gram-Schmidt algorithm to efficiently compute orthogonal projections.
- Reinterpret certain optimisation problems in terms of orthogonal projections in order to solve them.
- Understand the determinant as the mathematical notion encapsulating the idea of (algebraic) volume. Calculate the determinant of $2 \times 2$ and $3 \times 3$ matrices using standard formulas, and calculate higher-dimensional ones via Gaussian elimination.
- Compute the characteristic polynomial, eigenvalues, and eigenvectors of a square matrix. Determine whether a given square matrix is diagonalisable, and if so find a diagonalising matrix.
- Study the asymptotic behaviour of discrete dynamical systems by diagonalising (if possible) the associated matrix.
- Use finite Markov chains to model certain processes, or prove that a given process admits a unique stable state.
- Understand the mathematics behind the PageRank algorithm.