Aims

The course aims to provide an introduction to the calculus for functions of several variables, which will provide sufficient expertise for use in various later courses. The students will also develop their general skills in differentiation, integration and algebraic manipulation.

Syllabus

**Partial differentiation:** Functions of several variables, partial derivatives and higher order partial derivatives. Matrix ‘total’ derivatives. The chain rule, implicit differentiation. (9 lectures)

**Applications of partial differentiation:** Taylor expansions, tangent planes, maxima and minima, Lagrange multipliers. The inverse function theorem (in 1 and 2-dimensions). (7 lectures)

**Integration:** Double integrals, interchange of order of integration. Change of variables. Polar coordinates. Triple integrals. (8 lectures)

**Applications of integration:** Areas, volumes. (3 lectures)

**Integrals over infinite regions:** The definition of the convergence of integrals of functions on unbounded intervals. Comparison tests and absolute convergence tests of integrals. (3 lectures)

**Sequences:** Define a sequence of real numbers. Define bounded and convergent sequences, and the limit of a convergent sequence. (3 lectures).

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** up to 30% by class tests or other continuous assessment at least 70% by end of course 2-hour exam

**Resit Type:** exam only
By the end of the course, students should be able to:

- work out partial derivatives of any order.
- apply the chain rule to functions of functions.
- work out derivatives of implicitly defined functions.
- calculate Taylor expansions of functions of two variables.
- find extremal points of functions of two variables and determine whether they are maxima or minima.
- find extremal points of constrained max/min problems using Lagrange multipliers.
- evaluate double integrals over various planar regions and triple integrals over simple volumes.
- interchange the order of integration of repeated integrals.
- change variables in multiple integrals. In particular, use polar coordinates to evaluate multiple integrals.
- use multiple integration to work out areas and volumes of simple geometrical objects.
- recall the definition of and investigate the convergence of integrals of functions on unbounded intervals (and in particular know the values of $p$ for which the standard integral $\int_1^\infty \frac{1}{x^p} \, dx$ converges)
- state and use the comparison and absolute convergence tests for integrals of functions on unbounded intervals.
- recall the definition of and investigate the convergence of integrals of unbounded functions on bounded regions (and in particular know the values of $p$ for which the standard integral $\int_0^1 \frac{1}{x^p} \, dx$ converges)
- state and use the comparison and absolute convergence tests for integrals of unbounded functions on bounded regions.