F1.7CB2
Calculus B

Aims

This module builds on the differential and integral calculus introduced in Calculus A, before moving on to introduce the basics of mathematical modelling techniques using first and second order ordinary differential equations.

The module develops integration methods such as integration by parts and reduction formulae and describes some applications of integration.

Solution methods for first and second order differential equations are introduced and used to investigate various physical problems.

Syllabus

Applications of differentiation: Tangents and normals, maxima and minima, curve sketching, Rates of change, L'Hopital's rule, approximations. (4 lectures)

Advanced Integration: Integration of rational and surd functions, integration by substitution, standard substitutions, integrals involving trig and hyperbolic functions, integration by parts, reduction formulae. More general areas under a curve, convergence of integrals. (8 lectures)

Differential Equations: First order differential equations: Variables separable, Linear, homogeneous. Second order differential equations: Solution of linear equations with constant coefficients, Solution of the homogeneous equation, Particular integrals and solution of the nonhomogeneous equation, Initial value problems. (8 lectures)


Modelling through second order equations: Newton's laws of motion: Projectiles in 1D, Falling bodies with air resistance, Motion under constant acceleration. Amplitude and Period of Simple Harmonic Motion, Damped oscillations. (4 lectures)

Recurrence relations: Introduction to the use of first-order and linear second-order recurrence relations in mathematical modelling (3 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: up to 20% by class tests or other continuous assessment 80% by end of course 2-hour exam
Resit Type: exam

Content: Jan2016
By the end of the course, students should be able to:

- compute the equation of tangents and normals to curves.
- determine the nature of stationary points using the first and second derivative tests.
- calculate limits as \( x \to \pm \infty \).
- determine the asymptotes for a curve.
- sketch a curve which may have turning points and asymptotes.
- apply the chain rule to solve problems involving related rates of change.
- understand how to do integration by substitution.
- evaluate the integrals of trig and hyperbolic functions.
- use integration by parts.
- obtain reduction formulae using integration by parts.
- solve first order differential equations which are separable, linear or homogeneous.
- find the general solution of homogeneous second order differential equations with constant coefficients.
- find particular integrals for nonhomogeneous second order differential equations with constant coefficients.
- find the general solution of nonhomogeneous second order differential equations with constant coefficients.
- solve initial value problems involving second order differential equations with constant coefficients.
- develop the first order differential equation to model physical situations involving linear growth or decay.
- determine the solution of first order differential equation models for various applications with given conditions and to use the solution to find values of any parameters involved.
- solve the logistic equation for modelling applications involving nonlinear growth and decay.
- interpret the solutions of differential equation models.
- know the connection between the position, velocity and acceleration of a particle.
- solve problems on projectiles in 1D which involve numerical data, symbols and air resistance.
- find the time to greatest height, the greatest height, the time of flight, for a projectile in 1D.
- derive and solve the equation for simple harmonic motion (SHM).
- determine the period and amplitude for a simple harmonic motion.
- apply solutions for harmonic motion to problems with given initial conditions and to use the particular solutions to answer various questions on the motion.
- derive solutions for damped oscillators.
- solve first-order and linear second-order recurrence relations.