Aims

This module provides a first course on differential calculus with an introduction to sequences and series. It is designed for students who will specialize in mathematics, actuarial mathematics or statistics. The module builds on what the students learned at school but provides a greater depth of study and introduces new material and concepts from real analysis.

Syllabus

Functions: Functions, domains and ranges, \( (2 \text{ lectures}) \).

Limits of functions: Limit of a function at a point, evaluation of limits, continuous functions. \( (6 \text{ lectures}) \).

Introduction to Differentiation: Differentiable functions, laws of differentiation, differentiation of standard functions, rules of differentiation. Leibniz theorem. \( (4 \text{ lectures}) \).

Inverse Functions: One-to-one and onto functions, inverse functions, inverse trigonometric functions, exponential functions, logarithms. \( (2 \text{ lectures}) \).

Advanced Differentiation: Parametric differentiation, implicit differentiation. Derivatives of inverse functions, inverse trigonometric functions. Hyperbolic functions and hyperbolic equations, derivatives for hyperbolic functions and inverse hyperbolic functions. L'Hopital's rule. \( (9 \text{ lectures}) \).

Sequences and Series: Limits of a sequence, Convergence of sequences, partial sums of sequences. Convergence of infinite series, tests for convergence of infinite series, power series, radius of convergence, Maclaurin series, sums of Maclaurin series. \( (8 \text{ lectures}) \).

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 20% by class tests or other continuous assessment
80% by end of course 2-hour exam
Resit Type: exam
By the end of the course, students should be able to:

- recognise domains and ranges for functions.
- understand the concept of limit of a function.
- calculate the limit of a function.
- understand the idea of a continuous function.
- define the derivative of a function as a limit.
- determine the derivative of functions from first principles.
- apply the basic rules of differentiation, namely the sum rule, the rule for multiplication by a scalar, the product rule, the quotient rule and the chain rule.
- state the standard derivatives for powers, trigonometric functions and exponential and logarithmic functions.
- calculate second and higher derivatives.
- calculate limits using l’Hopital’s rule.
- determine if functions are one-to-one or onto.
- understand the concept of the inverse of a function.
- understand the concept of the exponential function and logarithms.
- find first and second derivatives for curves given in parametric form.
- understand and apply implicit differentiation techniques.
- calculate the derivatives of the inverse trigonometric functions.
- define the hyperbolic functions and compute their derivatives.
- define the inverse hyperbolic functions and compute their derivatives.
- understand the concept and compute the limit of a sequence.
- compute formulas for the sum to $n$ terms of a sequence.
- compute the limits, if they exist, of some simple infinite series.
- use the comparison test, ratio test and absolute convergence test, to check convergence of series.
- find and use the Maclaurin series of functions.