Aims

The course aims at providing fundamental notions on the theory of solitons. Basic solitonic solutions of certain PDEs, such as the Korteweg-de Vries (KdV), non-linear Schrödinger and sine-Gordon equations are examined. The construction of one soliton and multi-soliton solutions is presented. The Lax pair formulation, the infinite conservation laws as well as the associated linear auxiliary problem are also discussed.

Syllabus

The Korteweg-de Vries (KdV) equation: Preliminaries and examples, travelling wave solutions, solitary waves. (4 lectures)

Scattering and inverse scattering: Introduce the scattering and inverse scattering methodologies. Present various relevant examples. Derive and solve the Marchenko equation. Construction of one soliton and multi-soliton solutions in KdV equation. (5 lectures)

Conservation laws, the Lax pair formulation and generic methods: Derive an infinity of conservation laws. Introduce the Lax pair for the KdV hierarchy. Introduce the Bäcklund transformation and Hirota’s method for obtaining solitonic solutions. (7 lectures)

General Inverse methods: The AKNS hierarchy. The 2x2 eigenvalue problem. Time evolution of the scattering data. More examples: the non-linear Schrödinger (NLS) and sine-Gordon models. (7 lectures)

More on Lax pairs and conservation laws: The auxiliary linear problem. Conserved quantities for the NLS and sine-Gordon models. More examples on integrable PDEs and difference equations (Toda chain, discrete NLS). (7 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment
100% by end of module 2-hour exam
Resit Type: exam for MSc

Content: July 2015
By the end of the course, students should be able to:

- derive simple solitary wave solutions of certain PDEs such as the Korteweg-de Vries (KdV) equation
- understand the fundamental methods of scattering and inverse scattering, and solve the associated Marchenko equation
- derive solitonic solutions from scattering data and vice versa
- construct one soliton and multi-soliton solutions for various PDEs. e.g. KdV, NLS, sine-Gordon using Hirota's method and Bäcklund transformations
- derive the associated infinite conservation laws, and apply the Lax pair formulation on certain integrable PDEs
- study and apply more general inverse methods, i.e. the AKNS hierarchy. Derive the time evolution of scattering data.
- solve problems using the auxiliary linear formalism. Systematically extract the associated conserved quantities
- work out fundamental examples of integrable PDEs such as the KdV, NLS and sine-Gordon models as well as some discrete models e.g. the Toda chain, discrete NLS