Aims

The aims of this module are to develop understanding of continuous random variables and numerical simulation. We will examine Brownian motion and its properties and develop stochastic integrals and stochastic calculus. This will be done in a practical way with numerical simulations underpinning the analysis. We will introduce numerical methods for SDEs and the different notions of convergence for numerical methods. We will analyse convergence of Euler–Maruyama method. Monte-Carlo simulations and convergence will also be discussed. Typical example SDEs in the course are Langevin equations, Geometric Brownian motion and Ornstein-Uhlenbeck process.

Syllabus

Introduction to random number generation and Monte-Carlo simulations: (6 lectures)

Brownian motion, scaling property, non-differentiability and white noise. Numerical approximation. Fourier expansion and coloured noise.: (9 lectures)

Ito and Stratonovitch Integral, SDEs and Numerics: (15 lectures)

Teaching and Assessment

Contact Hours: 4 hours per week.
Assessment: 25% by class tests or other continuous assessment
75% by end of course 2-hour exam
Resit Type: exam for MSc/PG Diploma
By the end of the course, students should be able to:

- Demonstrate understanding of application of Monte-Carlo simulation and variants
- Demonstrate knowledge of Brownian motion and properties and ability to simulate numerically.
- Understand difference between Ito and Stratonovich integrals.
- Demonstrate knowledge of basic properties of an Ito integral.
- Understand SDE as an integral equation.
- Be able to apply Ito formula to solve standard one dimensional SDE.
- Demonstrate understanding of numerical approaches to SDEs, derive and implement Euler-Maruyama scheme.
- Demonstrate awareness of difference in weak and strong convergence.
- Understand the key elements in strong convergence of the EM method.