Aims

This course develops methods of multidimensional calculus to investigate geometrical properties of smooth curves and surfaces.

Syllabus

**Curves in Euclidean space:** Definition and symmetry of 3-dimensional Euclidean space, parametrised curves in Euclidean space, arc length, curvature and torsion. (7 lectures)

**Vector fields and differential forms:** Vector fields as derivative operations, differential 1-forms, line integrals, forms of higher degree, exterior derivative, relation to vector calculus operations div, grad and curl. (6 lectures)

**Moving frames and structure equations:** Definition of a moving frame in Euclidean space, connection forms, first and second structure equations. (4 lectures)

**Surfaces in Euclidean space:** Surfaces described by maps into Euclidean space, normal vectors and tangent vectors, adapted frames, first and second fundamental forms, Gauss and mean curvature, Gauss and Codazzi equation. (6 lectures)

**Curvature and geodesics:** The meaning of curvature, Theorema Egregium, definition of geodesic, introduction to Riemannian geometry. (7 lectures)

**Private Reading:** Learn assigned reading material which will investigate topics related to the course, but be beyond the scope of material taught in lectures. (0 lectures)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week
**Assessment:** 0% by class tests or other continuous assessment
100% by end of course 3-hour exam
**Resit Type:** exam for MSc
By the end of the course, students should be able to:

- compute arc lengths, curvature and torsion of curves and be able to interpret results geometrically
- compute action of vector field on a function and pairing of vector field with a 1-form
- compute wedge products and exterior derivatives of differential forms
- understand notion of moving frame and compute connection forms for given frame
- understand description of surface in terms of maps into Euclidean space
- compute normal and tangent vector fields to a parametrised surface
- understand definition of first and second fundamental form and of principal curvatures
- compute Gauss and mean curvature for simple parametrised surfaces
- understand geometrical meaning of Gauss curvature
- derive equation for geodesic on a given parametrised surface and find geodesics in simple cases
- recall major theorems and definitions; understand the proofs of the theorems; reproduce proofs of minor results; reproduce major results when provided guidance