Aims

This course develops methods of multidimensional calculus to investigate geometrical properties of smooth curves and surfaces.

Syllabus

Curves in Euclidean space: Definition and symmetry of 3-dimensional Euclidean space, parametrised curves in Euclidean space, arc length, curvature and torsion. (7 lectures)

Vector fields and differential forms: Vector fields as derivative operations, differential 1-forms, line integrals, forms of higher degree, exterior derivative, relation to vector calculus operations div, grad and curl. (6 lectures)

Moving frames and structure equations: Definition of a moving frame in Euclidean space, connection forms, first and second structure equations. (4 lectures)

Surfaces in Euclidean space: Surfaces described by maps into Euclidean space, normal vectors and tangent vectors, adapted frames, first and second fundamental forms, Gauss and mean curvature, Gauss and Codazzi equation. (6 lectures)

Curvature and geodesics: The meaning of curvature, Theorema Egregium, definition of geodesic, introduction to Riemannian geometry. (7 lectures)

Private Reading: Learn assigned reading material which will investigate topics related to the course, but be beyond the scope of material taught in lectures. (0 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment
100% by end of course 3-hour exam
Resit Type: exam for MSc
By the end of the course, students should be able to:

- compute arc lengths, curvature and torsion of curves and be able to interpret results geometrically
- compute action of vector field on a function and pairing of vector field with a 1-form
- compute wedge products and exterior derivatives of differential forms
- understand notion of moving frame and compute connection forms for given frame
- understand description of surface in terms of maps into Euclidean space
- compute normal and tangent vector fields to a parametrised surface
- understand definition of first and second fundamental form and of principal curvatures
- compute Gauss and mean curvature for simple parametrised surfaces
- understand geometrical meaning of Gauss curvature
- derive equation for geodesic on a given parametrised surface and find geodesics in simple cases
- recall major theorems and definitions; understand the proofs of the theorems; reproduce proofs of minor results; reproduce major results when provided guidance