Aims

This module provides an introduction to the derivation and analysis of techniques for the numerical approximation of ordinary differential equations. The theory is reinforced through practical project work.

Syllabus

**Introduction and Basic Concepts:** definitions, reduction to first order system of ODEs, Euler’s method, global and local truncation errors (LTE), convergence (5 lectures)

**Linear Multistep Methods:** derivation, LTE, implicit vs. explicit, linear difference equations, zero and absolute stability, stiffness, backward difference formulae (12 lectures)

**Runge Kutta Methods:** derivation of explicit schemes, stability, efficiency, vector-valued problems (5 lectures)

**Time Step and Error Control:** error estimates based on two different methods, efficient use of RK schemes in error control (3 lectures)

**Final Topic:** Boundary value problems or geometric integration (3 lectures)

**Practical Work:** (3 lectures)

**Revision and problem solving:** (3 lectures)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial/computer lab per week

**Assessment:** 25% by class tests or other continuous assessment
75% by end of module 2-hour exam

**Resit Type:** exam for MSc
By the end of the course, students should be able to:

- reduce higher order systems to a first order system of ODEs
- derive Euler's method and use it to approximate ODEs
- prove convergence of Euler's method
- (RK) derive explicit two-stage RK schemes
- (RK) establish stability intervals for explicit RK schemes
- (RK) investigate the computational efficiency of RK schemes
- (RK) use explicit RK schemes to approximate scalar and vector-valued examples
- derive error estimates based on the use of two different schemes
- combine RK schemes for efficient error control
- (LMM) derive simple LMMs via integration of Lagrange interpolants
- (LMM) deal with starting values in LMMs
- (LMM) carry out LTE analysis for general and specific LMMs
- (LMM) derive LMMs via LTEs
- (LMM) understand the relative advantages and disadvantages of explicit and implicit methods
- (LMM) deal with implicit methods
- solve homogeneous and simple inhomogeneous linear difference equations
- (LMM) carry out zero stability analysis of LMMs
- (LMM) understand the relationships between zero stability, convergence, LTE, root=1
- (LMM) carry out absolute stability analysis of LMMs (scalar and vector-valued problems)
- (LMM) deal with nonlinear systems of equations
- (LMM) understand the concept of stiffness
- (LMM) understand the construction, properties and uses of backward difference formulae (BDFs)
- deal with the ideas and methods in the final topic
- understand and use the Lax Equivalence Theorem
- write a computer programme to implement fixed step explicit RK and explicit LMMs for general ODE systems, and, for linear systems, implement schemes such as the Trapezoidal rule