Aims

This module provides an introduction to the derivation and analysis of techniques for the numerical approximation of ordinary differential equations. The theory is reinforced through practical project work.

Syllabus

Introduction and Basic Concepts: definitions, reduction to first order system of ODEs, Euler’s method, global and local truncation errors (LTE), convergence (5 lectures)

Linear Multistep Methods: derivation, LTE, implicit vs. explicit, linear difference equations, zero and absolute stability, stiffness, backward difference formulae (12 lectures)

Runge Kutta Methods: derivation of explicit schemes, stability, efficiency, vector-valued problems (5 lectures)

Time Step and Error Control: error estimates based on two different methods, efficient use of RK schemes in error control (3 lectures)

Final Topic: Boundary value problems or geometric integration (3 lectures)

Practical Work: (3 lectures)

Revision and problem solving: (3 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial/computer lab per week
Assessment: 25% by class tests or other continuous assessment
75% by end of course 2-hour exam
Resit Type: exam for MSc
By the end of the course, students should be able to:

- reduce higher order systems to a first order system of ODEs
- derive Euler's method and use it to approximate ODEs
- prove convergence of Euler's method
- (RK) derive explicit two-stage RK schemes
- (RK) establish stability intervals for explicit RK schemes
- (RK) investigate the computational efficiency of RK schemes
- (RK) use explicit RK schemes to approximate scalar and vector-valued examples
- derive error estimates based on the use of two different schemes
- combine RK schemes for efficient error control
- (LMM) derive simple LMMs via integration of Lagrange interpolants
- (LMM) deal with starting values in LMMs
- (LMM) carry out LTE analysis for general and specific LMMs
- (LMM) derive LMMs via LTEs
- (LMM) understand the relative advantages and disadvantages of explicit and implicit methods
- (LMM) deal with implicit methods
- solve homogeneous and simple inhomogeneous linear difference equations
- (LMM) carry out zero stability analysis of LMMs
- (LMM) understand the relationships between zero stability, convergence, LTE, root=1
- (LMM) carry out absolute stability analysis of LMMs (scalar and vector-valued problems)
- (LMM) deal with nonlinear systems of equations
- (LMM) understand the concept of stiffness
- (LMM) understand the construction, properties and uses of backward difference formulae (BDFs)
- deal with the ideas and methods in the final topic
- understand and use the Lax Equivalence Theorem
- write a computer programme to implement fixed step explicit RK and explicit LMMs for general ODE systems, and, for linear systems, implement schemes such as the Trapezoidal rule