Aims

Problems in optimization are the most common applications of mathematics. The main aim of this course is to present different methods of solving optimization problems in the three areas of linear programming, nonlinear programming, and classical calculus of variations. In addition to theoretical treatments, there will be some introduction to numerical methods for optimization problems.

Syllabus

Introduction: A survey of some simplified examples of common real world situations leading to optimization problems. Basic formulation and theory of optimization problems (3 lectures)

Linear programming: Linear programming (optimisation of linear functions subject to linear constraints): basic theory; simplex method; duality, practical techniques. (7 lectures)

Nonlinear programming: Nonlinear programming (optimisation of nonlinear functions subject to constraints): Lagrange multipliers, Karush-Kuhn-Tucker optimality conditions, convexity, duality. (7 lectures)

Approximation methods for Nonlinear programming: Line search methods, gradient methods, conjugate gradient methods. (4 lectures)

Calculus of variations: Euler-Lagrange equation, boundary conditions. (9 lectures)

Revision and problem solving: (3 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial/computer lab per week
Assessment: 0% by class tests or other continuous assessment
100% by end of module 3-hour exam
Resit Type: exam for MSc
By the end of the course, students should be able to:

- understand the broad classification of optimization problems, and where they arise in simple applications.
- understand the concept of an objective function, a feasible region, and a solution set of an optimization problem.
- write down the dual linear programming problem.
- use the simplex method to find an optimal vector for the standard linear programming problem and the corresponding dual problem.
- prove the optimality condition for feasible vectors for LP and DLP.
- use Lagrange multipliers to solve nonlinear optimization problems.
- write down and apply Kuhn-Tucker conditions for constrained nonlinear optimization problems.
- understand the importance of convexity in nonlinear optimization problems.
- apply basic line search methods to one-dimensional optimization problems.
- apply gradient methods to optimization problems.
- apply conjugate gradient methods to optimization problems.
- apply approximate methods for constraint problems.
- derive the Euler-Lagrange equation.
- use the Euler-Lagrange equation to solve the classical problem of calculus of variations.

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