Dynamical Systems

2017/18

Aims

The course aims to introduce the main features of dynamical systems, particularly as they arise from systems of ordinary differential equations as models in applied mathematics. The topics presented will include phase space, fixed points and stability analysis, bifurcations, Hamiltonian systems and dissipative systems. Discrete dynamical systems will also be discussed briefly, leading to the idea of a 'chaotic' dynamical system.

Syllabus

An Introduction to Dynamical Systems: Background and examples, dynamical systems, attractors and invariant sets. *(2 lectures)*

Phase Portraits: Phase portraits in 1D, topological equivalence, linear systems, linear 2D systems, stability and linearization of non-linear systems, Lyapunov stability, drawing global phase portraits. *(7 lectures)*

Local bifurcations: The Implicit Function Theorem, classification of bifurcations in 1D, bifurcations in higher dimensions. *(6 lectures)*

Conservative and Hamiltonian Systems: Conservative forces, finite-dimensional Hamiltonian systems, symplectic structure, conservation laws, Liouville's Theorem, Poincaré's Recurrence Theorem, stability of fixed points. *(5 lectures)*

Dissipative Systems: Absorbing sets, dissipative systems and global attractors, structure of the global attractor, proving dissipativity, gradient systems. *(6 lectures)*

Discrete Dynamical Systems and chaos: Definition of a discrete dynamical system, graphical analysis of $1D$ discrete dynamical systems, stability of fixed points and periodic orbits, chaotic orbits – definition and examples. *(4 lectures)*

Teaching and Assessment

Contact Hours:	3 lectures and 1 tutorial per week
Assessment:	0% by class tests or other continuous assessment
	100% by end of course 2-hour exam
Resit Type:	exam for MSc

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By the end of the course, students should be able to:

- describe the main features of dynamical systems and their realisation as systems of ordinary differential equations
- identify fixed points of simple dynamical systems, and study the local dynamics around these fixed points, in particular to discuss their stability and bifurcations
- use a range of specialised analytical techniques which are required in the study of dynamical systems
- describe dynamical systems geometrically and represent them graphically via phase plane analysis
- understand and predict the occurrence and consequences of bifurcations
- explain and prove special properties of finite-dimensional Hamiltonian systems, in particular conservation laws, Liouville's Theorem and Poincaré's Recurrence Theorem
- prove simple theoretical results about abstract dynamical systems
- understand the origin of dissipation and its effect on the orbits of dynamical systems
- find fixed points and period orbits of discrete dynamical systems, and find their stability
- do graphical analysis of $1D$ discrete dynamical systems
- understand the basic properties of a chaotic dynamical system

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