Aims

This course will teach the application of mathematical models to a variety of problems in biology and medicine. It will show the application of ordinary differential equations to simple biological and medical problems, the use of mathematical modelling in biochemical reactions, the application of partial differential equations in describing spatial processes such as cancer growth and pattern formation in embryonic development, and the use of delay-differential equations in physiological processes.

Syllabus

**ODE models in biology and medicine:** Introduction to mathematical modelling using ODE models; bacterial growth; growth in a chemostat; tumour-immune system dynamics; neural modelling and the Fitzhugh-Nagumo equations; revision of phase plane methods and non-dimensionalisation techniques. (5 lectures)

**Reaction kinetics:** the Law of Mass Action; modelling enzymatic reactions, including co-operative behaviour and substrate inhibition; analysis of a simple enzymatic reaction; pseudo-steady state hypothesis; matched asymptotics and singular perturbation theory; biological oscillators and demonstration of limit cycles in a simple model using the Poincare-Bendixson theorem; enzyme production. (7 lectures)

**Biological movement and pattern formation:** modelling cell movement; examples of patterning in biology, for example animal coat markings and bacteria patterns; the Turing mechanism as a model for pattern formation and the conditions for diffusion driven instability; patterns on one and two dimensional finite domains and applications to animal pigmentation; chemotaxis as a model for pattern formation. (7 lectures)

**Travelling waves:** reaction diffusion equations and their applications to wound healing, cancer growth, epidemiology; the Fisher equation - travelling waves and derivation of the wave speed; cubic kinetics; travelling waves for multiple populations and applications to epidemiology. (7 lectures)

**Delay differential equations:** Introduction to delay differential equations in modelling; derivation of a critical delay for stability in a single DDE. Construction of periodic solutions for piecewise constant negative feedback. Applications to modelling in physiological processes, for example Cheynes-Stokes breathing, hematopoietic regulation, testosterone secretion. (4 lectures)

**Revision and problem solving:** (3 lectures)
Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment
             100% by end of course 3-hour exam
Resit Type: exam for MSc
By the end of the course, students should be able to:

- Interpret the biological significance of terms in mathematical models of biology and medicine.
- Apply non-dimensionalisation techniques.
- Determine steady states, their stability and produce phase plane portraits.
- Understand the meaning of an excitable system and perform simple phase plane analysis of Fitzhugh-Nagumo type models.
- Understand the Law of Mass Action and apply it to modelling simple chemical reactions.
- Derive appropriate equations for a variety of enzymatic reactions.
- Understand the use of a pseudo-steady-state hypothesis in chemical reactions.
- Employ singular perturbation techniques for models of simple enzymatic reactions.
- Demonstrate oscillatory behaviour in simple biochemical models.
- Understand the role and modelling of spatial movement terms in biological models.
- Understand the process of diffusion-driven instability.
- Derive general DDI conditions for a Turing model and use them to obtain parameter domains for specific kinetics.
- Use the DDI conditions to determine which patterns will grow on one-dimensional and two-dimensional finite domains.
- Determine conditions for aggregation in models for chemotaxis.
- Understand the way in which phase plane methods are used to study travelling wave solutions of scalar reaction-diffusion.
- Understand the key differences between travelling wave solutions of scalar reaction-diffusion equations with quadratic or cubic kinetics.
- Derive travelling wave speeds for standard-type scalar reaction-diffusion equations, and derive wave forms also in simple cases.
- Understand the concept of a delay differential equation and the use of the delay as a bifurcation parameter, including calculation of the critical delay for stability.
- Construct explicit periodic solutions for a DDE with piecewise constant feedback.