Aims

The goal of the course is to derive the mathematical models for fluid mechanics and use them to explain natural phenomena as well as outline important engineering applications.

Syllabus

Introduction: Continuum hypothesis; trajectories and streamlines; continuity equation; incompressible flow, stream functions; material derivative and transport theorem.

Derivation of the Euler and Navier-Stokes equations: internal forces, stress tensor, stress/strain relation and Newtonian/Non-Newtonian fluids; equation of state and energy considerations; vorticity.

Inviscid flow: Bernoulli’s theorem; vortex lines, tubes and Kelvin’s circulation theorem, incompressible, irrotational/potential flow; use of complex variables; flows corresponding to singularities; method of images; Blasius’s theorem, conformal mappings and aerofoils.

Viscous flow: example flows—Couette, Poiseuille, laminar, flows down flat inclines; Non-dimensionalization and Reynolds number; Stokes flow; Lubrication theory; boundary layer theories; vortex sheets; Euler and Prandtl limits.

Compressible flow: characteristics, shocks, flow driven by pistons; hyperbolic conservation laws, Riemann problem.

Revision by means of problem solving:

Additional reading: an additional reading topic on Mathematical Fluid Mechanics.

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment

100% by end of module 3-hour exam

Resit Type: exam for MSc only
By the end of the course, students should be able to:

- Understand the continuum hypothesis, and know how to derive particle trajectories and streamlines from a given velocity field.
- Know what incompressibility means and its implications for a given flow.
- Derive the material derivative and prove the transport theorem.
- Demonstrate internal and external forces on a fluid parcel, how they arise, and distinguish internal pressure forces from shear forces.
- Derive the Euler and Navier-Stokes equations.
- Appreciate how to close the system of equations via incompressibility or an equation of state and a pressure/density relation.
- Know the context of Bernoulli's theorem and derive it, solve hydrostatic problems using it.
- Understand fluid rotation and vorticity and its consequences.
- Appreciate the natural abundance of vortex lines and tubes nature.
- State Kelvin's circulation theorem.
- Solve simple well-known incompressible and irrotational/potential flows.
- Know how to use complex variables in “two-dimensional” flows and their applications to aerofoils.
- Understand the effect of viscosity, and solve many simple flows.
- Be able to non-dimensionalize the Navier–Stokes equations, derive the Reynolds number, and appreciate its significance in modelling and simulation.
- Know what a Stokes flow is and how to solve simple examples.
- Understand the basics behind lubrication theory and boundary layer theories.
- Solve compressible flows, for example Burgers type PDEs, via characteristics.