Aims

The goal of the course is to derive mathematical models for fluid mechanics, to use them for the explanation of natural phenomena and to outline important engineering applications.

Syllabus

**Introduction:** continuum hypothesis; trajectories and streamlines; continuity equation; incompressible flow; stream functions; material derivative and transport theorem.

**Derivation of the Euler and Navier-Stokes equations:** internal forces, stress tensor, stress/stretch relation and Newtonian/Non-Newtonian fluids; equation of state and energy considerations; vorticity.

**Inviscid flow:** Bernoulli’s theorem; vortex lines, tubes and Kelvin’s circulation theorem, incompressible, irrotational/potential flow; use of complex variables for two-dimensional flows.

**Viscous flow:** example flows—Couette, Poiseuille, laminar, flows down flat inclines; Non-dimensionalization and Reynolds number; Stokes flow; Lubrication theory; boundary layer theories; vortex sheets; Euler and Prandtl limits.

**Revision:** solving problems related to the course material.

**Additional reading:** a reading topic in mathematical fluid mechanics.

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** 0% by class tests or other continuous assessment
100% by end of course 3-hour exam

**Resit Type:** exam for MSc only

Content: August 2018
By the end of the course, students should be able to:

- Understand the continuum hypothesis, and know how to derive particle trajectories and streamlines from a given velocity field.
- Know what incompressibility means and its implications for a given flow.
- Derive the material derivative and prove the transport theorem.
- Demonstrate internal and external forces on a fluid parcel, how they arise, and distinguish internal pressure forces from shear forces.
- Derive the Euler and Navier-Stokes equations.
- Appreciate how to close the system of equations via incompressibility or an equation of state and a pressure/density relation.
- Know the context of Bernoulli's theorem and derive it, solve hydrostatic problems using it.
- Understand fluid rotation and vorticity and its consequences.
- Appreciate the natural abundance of vortex lines and tubes nature.
- State Kelvin's circulation theorem.
- Solve simple well-known incompressible and irrotational/potential flows.
- Know how to use complex variables in simple two-dimensional flows.
- Understand the effect of viscosity, and solve simple flows.
- Be able to non-dimensionalize the Navier–Stokes equations, derive the Reynolds number, and appreciate its significance in modelling and simulation.
- Know what a Stokes flow is and how to solve simple examples.
- Understand the basics behind lubrication theory and boundary layer theories.

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