Aims

Solitons are localized travelling wave packets that maintain their shapes as they travel, and emerge unaltered after interacting with each other. Due to this “particle-like” nature solitons are of mathematical significance and have a plethora of realistic applications including optical fibres, water wave and tsunami formation, Bose-Einstein condensates and chemical reactions among others. In fact, the soliton is part of Heriot-Watt history being discovered by J. S. Russell who noticed a solitary “wave of transition” that travelled for great distance on the Union canal just 1 mile from campus. In this course we will develop the methods to study and understand solitons. To achieve this we will first introduce basic quantum mechanical notions and in particular Schrödinger’s equation and use this as a background to study non-linear systems that display soliton behaviour.

Syllabus

The Schrödinger equation: Introduce the time dependent and the time independent Schrödinger equation. Study fundamental quantum mechanical systems. (8 lectures)

Mathematical formalism: Introduce the mathematical formalism of quantum mechanics: linear algebra, function spaces and generalized statistical interpretation. (7 lectures)

The Korteweg-de Vries (KdV) equation: Preliminaries and examples, travelling wave solutions, solitary waves. (4 lectures)

Conservation laws, the Lax pair formulation and generic methods: Derive an infinity of conservation laws for KdV. Introduce the general Lax pair formulation and in particular for the KdV hierarchy. (4 lectures)

General Inverse methods: The AKNS hierarchy. The 2x2 eigenvalue problem. Introduce the Bäcklund transformation for obtaining soliton solutions. Typical examples: the non-linear Schrödinger (NLS) and sine-Gordon models. (7 lectures)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment
                      100% by end of course 2-hour exam
Resit Type: none
By the end of the course, students should be able to:

- derive the time dependent and time independent Schrödinger equation
- solve prototype fundamental quantum mechanical systems such as the harmonic oscillator, the infinite square well, finite potential wall etc.
- understand the mathematical formalism of quantum mechanics. Linear algebra, function spaces and generalized statistical interpretation of quantum mechanics.
- derive simple solitary wave solutions of certain PDEs such as the Korteweg-de Vries equation
- derive the associated infinite conservation laws, and apply the Lax pair formulation on certain integrable PDEs
- study and apply more general inverse methods, i.e. the AKNS hierarchy.
- apply the Bäcklund transformation method to obtain soliton solutions.