Aims

To give an introduction to some advanced topics in number theory.

Syllabus

**Basics:** Revision of basic number theory: Euler’s \( \phi \) function; Modular arithmetic; Fundamental Theorem of Arithmetic and Chinese Remainder Theorem  (3 lectures)

**Congruences and units mod \( n \):** Polynomial congruences; Hensel’s Lemma; algebraic structure of the group of units in \( \mathbb{Z}_n \); primitive units. (5 lectures)

**Quadratic residues:** The Legendre symbol and the law of quadratic reciprocity  (5 lectures)

**Distribution of primes:** Primes in specific congruence classes; Fermat primes and Mersenne primes; The Prime Number Theorem. (5 lectures)

**Sums of squares:** Gaussian integers; sums of squares. (5 lectures)

**Möbius inversion:** Arithmetic functions; the Möbius function and the Möbius inversion formula. (4 lectures)

**The Riemann zeta function:** Dirichlet series; introduction to the Riemann \( \zeta \)-function. (3 lectures)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial per week

**Assessment:** 0% by class tests or other continuous assessment

100% by end of course 2-hour exam

**Resit Type:** none
By the end of the course, students should be able to:

- have a good grasp of the fundamentals of number theory
- be able to manipulate Legendre symbols using quadratic reciprocity, and apply this to quadratic congruence problems
- have an understanding of the theory of the distribution of primes
- understand and manipulate Gaussian integers, and apply them to problems about sums of squares
- have an understanding of the basic theory of arithmetic functions and Dirichlet series