Aims

This module provides an introduction to the techniques and analysis required to find the numerical solution of partial differential equations using both finite difference and finite element approaches. The theory is reinforced through practical computing work.

Syllabus

**Finite difference approximations:** classification of PDE's; forward, backward and central differences, Taylor series. \((3 \text{ lectures})\)

**Parabolic PDEs:** finite difference approximation of the heat equation; local truncation error analysis; stability and convergence; multi-level and ADI schemes. \((12 \text{ lectures})\)

**Hyperbolic PDEs:** travelling wave solutions; comparison of schemes for the advection equation; second order wave equations; nonlinear conservation laws. \((9 \text{ lectures})\)

**Elliptic PDEs:** Introduction, simple finite difference scheme. Finite element method (FEM); variational formulation of the FEM; nodal basis functions and matrix elements. \((6 \text{ lectures})\)

**Revision and problem solving:** \((3 \text{ lectures})\)

Teaching and Assessment

**Contact Hours:** 3 lectures and 1 tutorial/computer lab per week

**Assessment:** 15% by class tests or other continuous assessment
85% by end of course 2-hour exam

**Resit Type:** none
By the end of the course, students should be able to:

- construct finite differences for approximating partial derivatives
- analyse the local truncation error using Taylor series expansions
- perform stability analysis and state the Lax equivalence theorem
- write finite difference schemes in matrix algebra form
- derive the Euler, backward Euler and \( \theta \)-methods for solving parabolic PDE's
- distinguish explicit and implicit schemes
- apply the fictitious point method for incorporating flux boundary conditions
- apply ADI schemes for treating parabolic PDE's with more than one space dimension
- perform phase lag analysis for numerical solution of hyperbolic PDE's
- derive the leapfrog, Lax-Wendroff and Crank-Nicolson schemes for solving the advection equation
- use nonlinear conservation laws to derive the switching and nonlinear Lax-Wendroff schemes
- understand the variational formulation of the finite element method
- calculate linear and bilinear nodal basis functions for the FEM
- use the basis functions to calculate matrix elements for the FEM
- write computer code to approximate the solution of PDEs