Aims

The course aims to provide basic knowledge in the theory of partial differential equations, to study main properties of the classical equations of Mathematical Physics, to discuss the notion of weak solutions for simplest nonlinear PDE’s and some applications to mathematical modelling.

Syllabus

Linear second order equations: classification and reduction to canonical form of linear second order equations; solution of Cauchy problems for hyperbolic equations by reduction to canonical form; Well posed problems for partial differential equations. \( (6 \text{ lectures}) \)

The wave equation: energy method and uniqueness; solution by “spherical means”; well posedness of initial value problem. \( (6 \text{ lectures}) \)

The heat equation: solutions using Gaussian kernel; uniqueness; maximum principle for heat equation; well posedness of initial value problem. \( (5 \text{ lectures}) \)

Laplace’s equation: basic properties of harmonic functions; maximum principle for boundary value problem; existence of solutions and well posedness of boundary value problems for Laplace’s equation; Green’s functions. \( (7 \text{ lectures}) \)

Nonlinear conservation laws: discontinuous solutions of conservation laws; jump condition; model of a traffic flow; uniqueness and the entropy condition; Cole-Hopf transformation. \( (6 \text{ lectures}) \)

Revision and problem solving: \( (3 \text{ lectures}) \)

Teaching and Assessment

Contact Hours: 3 lectures and 1 tutorial per week
Assessment: 0% by class tests or other continuous assessment
100% by end of course 2-hour exam
Resit Type: none

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By the end of the course, students should be able to:

- classify linear second order PDE's
- derive the equation for characteristics for linear second order PDE's
- find characteristics for linear second order PDE's
- reduce hyperbolic linear second order PDE to a canonical form
- solve the Cauchy problem for a hyperbolic linear second order PDE (by reduction to canonical form)
- prove uniqueness of the solution of a Cauchy problem for the wave equation (by the energy method)
- express the solution of a Cauchy problem for the wave equation through "spherical means" (in dimension 2 and 3)
- understand the notion of well-posedness
- demonstrate well-posedness of a Cauchy problem for the wave equation
- know the notion of the domains of influence and dependence
- analyse the structure of the domains of influence and dependence for the wave equation in dimension 1, 2 and 3
- demonstrate time reversibility and finite speed of propagation for the wave equation
- prove that a solution of an initial value problem for the heat equation can be expressed through a Gaussian kernel
- explain an infinite speed of propagation and time irreversibility for the heat equation
- use the error function to express the solution of an initial value problem for the heat equation in the case of piece wise constant initial data
- formulate and prove the maximum principle for the heat equation in a bounded domain
- formulate and prove the maximum principle for the heat equation in the whole space
- demonstrate well-posedness of a initial value problem for the heat equation
- use Green's formulae to analyse the basic properties of harmonic functions
- prove uniqueness for the solutions of the Dirichlet and Neumann boundary value problem (by the energy method)
- prove the maximum principle for the Laplace operator
- prove uniqueness of the solution of the Dirichlet boundary value problem for Laplace’s equation (using the maximum principle)
- demonstrate well-posedness of the Dirichlet boundary value problem for Laplace’s equation
- know the notion of Green’s function
- express the solution of a Dirichlet boundary value problem for Laplace’s equation through the Green’s function
- prove Poisson's formula for the Green's function for a ball
- prove Harnack's inequality
- know the notion of a characteristic line for a non-linear conservation law
- show that a smooth solution of a non-linear conservation law is a constant function along a characteristics line
- derive the equation for characteristics for a non-linear conservation law
- find the smooth solution of a non-linear conservation law for simple initial conditions
- demonstrate that intersections of characteristics lead to existence of non-smooth solutions
- know the notion of a weak solution of a non-linear conservation law
- derive the jump condition
- analyse the model of a traffic flow
- demonstrate non-uniqueness of weak solutions
- know the notion of the entropy solution
- use the Cole-Hopf transformation to reduce Burgers equation to the heat equation

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