Contact Tracing

Johannes Müller

Technical University Munich
Center for Mathematics
Contact Tracing - Aim

Concept:
If we find an infected individual (so-called “index case”), we ask for (potentially) infectious contacts. Contact Partners are called in to a doctor. In this way, it is possible to find more infectious persons.

Related Methods:
Clustering (“who has a similar behaviour like you” instead of “with whom did you have contact”), Screening

Diseases:
HIV, SARS etc., Tuberculosis, sexually transmitted diseases (Gonorrhea, Chlamydia)....
Contact Tracing - Questions

• What’s the effect of a contact tracing program?

• How is an efficient contact tracing procedure designed?
  – Do we trace as much as we can?
  – Do we trace only the contacts of the primary index case?
  – How hard do we search for contacts?

• How does contact tracing differs in the onset of an epidemic and an endemic state?

• Estimate Parameters?

• Can we find monitoring tools for contact tracing programs?
Contact Tracing - Some Models

(a) Hethcote/York, 1984: Gonorrhea
Deterministic model. Nonlinear correction of the incidence(!) function. No connection to stochastic models.

(b) Lourdes/Azoza, 1995: HIV
Deterministic model. Linear (!) incidence function. No connection to nonlinear stochastic models.

(c) Kretzschmar/Duynhjoven/Severijnen, 1996: Gonorrhea, Chlamydia
Detailed simulation model. No attempt for analysis.

(d) Huerta, Tsimring 2002
Stochastic network ("small world network") and mean field

(e) Fraser, Riley, Anderson, Ferguson 2004
Deterministic PDE-Model; Tracing process is handled in a linear way.

(f) Eames 2006
Pair approximation in stochastic network.
Basic Epidemic Model

Simple stochastic SIS model with random mixing.

Consider a population of $N$ individuals.

Define the standard epidemic model on this population:

- contact rate: $\beta$ (random mixing)
- probability $q$ for the transmission of the infection per S/I-contact
- recovery rate $\mu$
Contact Tracing - Different Situations

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT
Contact Tracing - Onset

- Onset of the diseases / initial phase of CT
- Onset of the diseases / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT

Graph of Infecteds:

**Nodes:** Infected and Infectious individuals.

**Edges:** Directed edge from “Infector” to “Infectee”.

**Tracing Process:**
- Define a screening rate: with rate $\sigma$ an individual is detected (the primary index case) and removed.
- Edges have the probability $p$ to become detected and to be removed.

**Neglected:** (many things, especially)
- contact tracing is no instantaneous process.
- no test is perfect.
- diseases are not Markovian processes.
- diseases are not homogeneous.
Contact Tracing - Onset

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT

Results in the Onset of the diseases / equilibrium phase of CT:

- The process tends to a stable exponentially growing phase, where distributions (e.g. size of connected components) stabilizes
- Formula for the effective reproduction number $R_e$ (though not nice in the general case)
  \[ E(\text{Number of detected cases per index case}) = p \frac{R_0}{1+R_0} + \mathcal{O}(p^2) \]
  where $p$ is the tracing probability
- $E(\text{size of a connected component in the infection graph}) \leq 1 + R_0$
- Structure of a connected component in the infection graph (without contact tracing)

(M', Kretzschmar, Dietz 2000; M', Möhle 2003; M', Hösel 2007)
Contact Tracing - Endemic Case

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT
Contact Tracing - Endemic Case

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT
Contact Tracing - Tracing Model in the Endemic State

Two different type of traced contacts:

- Infection Tracing
  Tracing of infectee or infector

- Random Tracing
  Tracing of contacts via that no infection has been transmitted.

Tracing Model:

- All contacts have to be considered.
- In a finite, randomly mixing populations, any person has had contact to all other persons (perhaps long ago).
- Declining of tracing probability with the time since contact is necessary to incorporate.
  \[ P(\text{tracing of a contact a time units ago}) = p_1 e^{-p_2 a} \]
- Detection/screening rate \( \sigma \).
Central tool:

$$\kappa(a) = P(\text{individual has been infected } a \text{ time units ago and is infectious since then})$$

$$\frac{d}{da} \kappa(a) = -\kappa(a)\{\mu + \sigma + \text{Removal rate via a traced contact}\}, \quad \kappa(0) = 1.$$
Contact Tracing in the Endemic state - Analysis

Central tool:

$$\kappa(a) = P(\text{individual has been infected } a \text{ time units ago and is infectious since then})$$

$$\frac{d}{da}\kappa(a) = -\kappa(a)\{\mu + \sigma + \text{Removal via a traced contact}\}, \quad \kappa(0) = 1.$$  

- Assumption: approximately constant prevalence.
- Ten cases (blue: infectious, black: susceptible):

![Diagram showing ten cases with blue and black lines representing infection and susceptibility over time.](image-url)
Contact Tracing in the Endemic state - Analysis

Good news: Up to the first order in $p_1$, the differential equation for $\kappa$ is possible to work out.

Bad news: The result is terribly lengthy.

Attempt: Approximation.
(a) neglect all contacts that took place before the last infectious period of an individual started
(b) Approximate terms in exponent by quadratic terms in time since infection $a$

Let

$$\tau_0 = \frac{q\beta}{(\mu + \sigma)} \quad \text{(Fraction of susceptibles without contact tracing)}$$

then

$$\kappa(a) \approx e^{-(\mu+\sigma)a} \frac{e^{-\sigma p_1 a}}{\text{No Screening}} \frac{e^{-\beta q \tau_0 p_1 \sigma a^2/2}}{\text{Infecter}} \frac{e^{-\beta (1-\tau_0) p_1 \sigma a^2/2}}{\text{Infectees}} + O(p_1^2) \quad \text{Random tracing: I-I}$$
Contact Tracing in the Endemic state - Results

Endemic Equilibrium:

\[ 1 = R_e = q \beta S \int_0^\infty \kappa(a) \, da \]

\[(\beta/(\mu + \sigma)) \in \{3, 4, 5\}, \, q = 0.4, \, \sigma/(\mu + \sigma) = 0.2\] 

Effect even for small values like \( p_1 \leq 0.1 \)
Contact Tracing in the Endemic state - Results

**Contact Tracing:**
Rough Estimation of scanned persons per index case (do not throw away these data!):

\[ E(\text{No of traced/scanned contacts per index case}) \approx p_1 R_0 / q \]

Rough Estimation of scanned persons per index case:

\[ E(\text{Detected Persons per Index Case}) \leq p_1 \left( \frac{1}{\text{Infector}} + \frac{1}{\text{Infectee}} + \frac{(1 - \tau_0) R_0}{q} \right) \]

Relative number of hits:

\[ \frac{\text{detected cases}}{\text{scanned cases}} \leq 2q / R_0 + (1 - \tau_0) \]

**Screening:**
Compare with the case of pure screening:

\[ \frac{\text{detected cases}}{\text{screened cases}} = (1 - \tau_0) \]

For \( R_0 \gg 1 \), the relative success (number of hits versus number of tests) is in the same range for contact tracing and screening; the latter is cheaper!
Contact Tracing in the Endemic state - Questions

Are there (endemic) situations where contact tracing may be of use?

- Core groups
  - Contact tracing may reveal a core group structure
    (a lot of detailed data is necessary - unrealistically?)
  - Contact tracing may reveal core group members
    (clustering may be more efficient in this)

- Asymptomatic cases
  - Contact tracing may reveal asymptomatic cases
Summary: Contact Tracing

- Different situations (onset/endemic state) require different mathematical methods and may lead to different conclusions!

Contact tracing in the Endemic State and a Random Mixing Population

- For large populations, we can analyse models for contact tracing in the endemic case.
- Contact tracing is able to reduce the prevalence of diseases even for small tracing probabilities.
- In a random mixing population, screening may be preferable.
- In a core group situation, clustering may be preferable.
- If you have many asymptomatic cases, contact tracing may be of use.