Some Absolutely Continuous Distributions

Name	Notation	p.d.f.	range	m.g.f.	mean	variance
Uniform	$U(a,b)$	$\frac{1}{b-a}$	$a \le x \le b$	$\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	$-\infty < x < \infty$	$e^{\mu\theta+\frac{1}{2}\sigma^2\theta^2}$	μ	σ^2
Gamma	$\Gamma(t,\lambda)$	$\frac{\lambda^t}{\Gamma(t)} x^{t-1} e^{-\lambda x}$	$0<x<\infty$, $t,\lambda>0$	$\left(\frac{\lambda}{\lambda-\theta}\right)^t$	$\frac{t}{\lambda}$	$\frac{t}{\lambda^2}$
Exponential	$\text{Exp}(\lambda)$	$\lambda e^{-\lambda x}$	$0<x<\infty$, $\lambda>0$	$\frac{\lambda}{\lambda-\theta}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Chi-square	χ^2_n	$\frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$	$0<x<\infty$, $n>0$ integer	$(1-2\theta)^{-n/2}$	n	$2n$
Beta	$\beta(\alpha_1,\alpha_2)$	$\frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} x^{\alpha_1-1}(1-x)^{\alpha_2-1}$	$0<x<1$, $\alpha_1,\alpha_2>0$	not useful	$\frac{\alpha_1}{\alpha_1+\alpha_2}$	$\frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_2)^2(\alpha_1+\alpha_2+1)}$
Cauchy	–	$\frac{\lambda}{\pi(\lambda^2+(x-\mu)^2)}$	$-\infty<x<\infty$, $\lambda>0$	doesn't exist but char. fn. is $e^{i\mu\theta-\lambda\lvert\theta\rvert}$	no moments exist	
Multivariate Normal	$N(\mu,\Sigma)$	$\frac{\exp\left\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right\}}{(2\pi)^{d/2}\det(\Sigma)^{1/2}}$	$x \in \mathbb{R}^d$	$e^{\mu^T\theta+\frac{1}{2}\theta^T\Sigma\theta}$	μ	covariance matrix Σ

Some Discrete Distributions

(All distributions below have integer-valued ranges)

Name	Notation	prob. fn.	range	m.g.f.	mean	variance
Binomial	$\text{Bin}(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	$0\le x\le n$, $0<p<1$	$[(1-p)+pe^\theta]^n$	np	$np(1-p)$
Poisson	$\text{Pois}(\lambda)$	$\frac{e^{-\lambda}\lambda^x}{x!}$	$0\le x<\infty$, $\lambda>0$ real	$e^{\lambda(e^\theta-1)}$	λ	λ
Hyper-geometric	$\text{Hgeo}(n,a,b)$	$\frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$	$0\le x\le n$, $a,b>0$ integers	not useful	$\frac{na}{a+b}$	$\frac{nab(a+b-n)}{(a+b)^2(a+b-1)}$
Negative binomial	$\text{Nbin}(k,p)$	$\binom{x-1}{k-1}p^k(1-p)^{x-k}$	$k\le x<\infty$, $0<p<1$	$p^k[e^{-\theta}-(1-p)]^{-k}$	k/p	$k(1-p)/p^2$
Geometric	$\text{Geo}(p)$	$p(1-p)^{x-1}$	$1\le x<\infty$, $0<p<1$	$p[e^{-\theta}-(1-p)]^{-1}$	$1/p$	$(1-p)/p^2$

<u>Multinomial:</u> multivariate extension of binomial

Prob. fn. is given by

$$f(x_1,x_2,\ldots,x_k) = \frac{n!}{x_!!x_2!\ldots x_k!x_{k+1}!} p_1^{x_1}p_2^{x_2}\ldots p_k^{x_k}p_{k+1}^{x_{k+1}}, \quad 0\le x_i\le n, \quad 0<p_i<1 \;\; \forall i=1,2,\ldots,n+1$$

where $p_1+p_2+\cdots+p_{k+1}=1$ and $x_1+x_2+\cdots+x_{k+1}=n$.

The m.g.f. is given by

$$[p_1e^{\theta_1}+p_2e^{\theta_2}+\cdots+p_ke^{\theta_k}+p_{k+1}]^n.$$

Moments: $E(X_i)=np_i$, $\text{Var}(X_i)=np_i(1-p_i)$, $\text{cov}(X_i,X_j)=-np_ip_j$, $i,j=1,2,\ldots,n+1$.