Insurance and queueing

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Abstract

There are strong relations between queueing and risk/insurance models. Consider, e.g., the surplus process $R_t = u + ct - \sum_{i=1}^{N_t} X_i$ of the classical Cramér-Lundberg model that describes the surplus at time t of an insurance portfolio, where u is the initial capital, c is a premium intensity and $\{X_i\}_{i \geq 1}$ is a sequence of i.i.d. positive claim sizes; the claim number process $\{N_t,\ t \geq 0\}$ is a Poisson process with intensity λ. It is well-known that the survival probability (i.e., 1 minus the ruin probability) of this process equals the steady-state probability that the workload in an $M/G/1$ queue with arrival rate λ and service times X_i is smaller than u.

In the first hour we consider an $M/G/1$ queue with the additional feature that the server works at speed r_1 (r_2) when the amount of work right after an arrival is smaller (larger) than some threshold K [2, 3]. We subsequently consider a related insurance model with different settings of the premium intensity (and of the claim intensity and claim size distribution) below and above some threshold K. We focus on one of several model variants from [4]: the level of the surplus process at arrival times of an independent Poisson observer determines which of the two parameter settings is chosen.

In the second hour we consider the Cramér-Lundberg risk model under the additional assumption that tax payments are deducted from the premium income (with a constant proportion $\gamma < 1$), whenever the free surplus is at a running maximum. By linking queueing concepts with risk theory, we derive a simple relation between the survival probabilities starting at some level u, for the modified surplus model ($\phi_\gamma(u)$) and the original surplus model ($\phi_0(u)$): $\phi_\gamma(u) = (\phi_0(u))^{1/(1-\gamma)}$. We finally extend this relation to the case of a tax rate that depends on the current surplus level [1].

References

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