Nonlinear filtering, stochastic PDEs and pathwise solutions

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We discuss classic problems of nonlinear filtering of partially observed diffusion processes, in particular the question of robustness of the filter. This leads us to studying the Cauchy problem for stochastic parabolic PDEs, driven by a multidimensional Wiener process W. We see that the problem of robustness of the filter can be formulated as a general question about the dependence of the solutions of stochastic parabolic PDEs on the trajectories of the driving process W.

Our theorem concerning the above question reads as follows: If W is approximated by continuous processes $\{W_n\}_{n=1}^{\infty}$ of bounded variation, such that almost surely W_n and their area processes converge in the uniform norm to W and its area processes, with some speed $n^{-\kappa}$, then the solutions of the corresponding PDEs, driven by W_n in place of W, converge almost surely, with essentially the same speed.

This theorem improves a result obtained jointly with Anton Shmatkov in 2005.

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