Logic and proof

Test 2 Friday 8th November 12.20 to 13.10

NAME: (please PRINT)

1

Circle one of the following:

MATHS CS OTHER

This test is worth 10% of your final grade. It is a closed book test. Full answers should be written in the spaces provided. University rules about cheating apply. The test is designed to last no more than 15 minutes but you can stay the whole 50 minutes if you wish. Show all working. **There are 4 questions.**

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1. Prove that $\neg(\neg p \vee (\neg q \vee \neg r))$ is logically equivalent to $p \wedge (q \wedge r)$ using truth tables. [3 marks]

p	q	r	¬(¬p ∨ (¬q ∨ ¬r))
F	F	F	**F**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**T**

[1 mark]

p	q	r	(p ∧ (q ∧ r))
F	F	F	**F**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**T**

[1 Mark]

Truth tables are the same so wff are logically equivalent. [1 Mark]

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2. Construct a wff in **disjunctive normal form** that has the following truth-table [2 marks].

p	q	r	A	
T	T	T	T	*
T	T	F	F	
T	F	T	F	
T	F	F	F	
F	T	T	T	*
F	T	F	F	
F	F	T	F	
F	F	F	T	*

$(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

[2 marks]

3. Construct a wff in **conjunctive normal form** that has the following truth-table [2 marks].

p	q	r	A	
T	T	T	T	
T	T	F	F	*
T	F	T	T	
T	F	F	F	*
F	T	T	T	
F	T	F	F	*
F	F	T	T	
F	F	F	F	*

$\neg A \equiv (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r)$

$\vee (\neg p \wedge \neg q \wedge \neg r)$

Thus $A \equiv (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r)$

$\wedge (p \vee q \vee r)$ [2 marks]

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4. Find a formula in implicational form which is logically equivalent to the following Horn formula

$$(p \vee \neg q) \wedge (\neg p \vee \neg q) \wedge (p).$$

[3 marks]

$(q \rightarrow p) \wedge (p \wedge q \rightarrow \underline{f}) \wedge (\underline{t} \rightarrow p)$

[1 mark for each correct formula]