Lecture 9

Question Construct a wff $XOR(p, q, r)$ which is T when exactly one of p, q, r is T and F otherwise.

Attempt 1 What about $p \oplus (q \oplus r)$?

We can **check** our answer using truth tables.

Let $p = T$, $q = T$, $r = T$.

Then $q \oplus r = F$ so $p \oplus (q \oplus r) = T$

Thus, this wff does not satisfy the conditions of the question.

Attempt 2 $(p \vee q \vee r) \wedge \neg(p \wedge q) \wedge \neg(p \wedge r)$
$\wedge \neg(q \wedge r)$

Check this using the truth table generator, and we find it works.

p	q	r	(p∨q∨r)∧¬(p∧q)∧¬(p∧r)∧¬(q∧r)
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T ✓
F	T	T	F
F	T	F	T ✓
F	F	T	T ✓
F	F	F	F

It is possible to generalize this result to obtain a wff that represents xor$(p_1, \ldots, p_n)$ [which is T when exactly one of $p_1, \ldots, p_n$ is T and F on all other occasions].

Section 1.5: PL in action

We shall show (by means of an example) that PL can be used as a kind of a programming language. However, not in the way that, say, Java, is a programming language. PL can be used to describe a problem P exactly ~~in such a way that~~ by means of a wff A in such a way that solutions to P correspond exactly to truth assignments satisfying A.

More generally, we have the following scheme for solving problems (which ones?) using PL:

Problem → Convert to wff P → SAT solver → Solutions to problem

- P describes the problem — it doesn't solve it. (P does not represent an algorithm)

- Solutions to problem correspond to the truth assignments that satisfy P.

[Full-size Sudoku case handled in this way (see exercises)].

<u>Example</u> We take a really simple "Sudoku" puzzle

Just two cells. Each cell can contain exactly one of the digits 1 or 2.

No number can be repeated in a row.

There are two obvious solutions to this puzzle:

1	2

and

2	1

.

I will now show how to represent the puzzle by means of a wff S, such that, where S is satisfiable corresponds exactly to the solutions to the puzzle. S <u>describes the problem</u>, it doesn't solve the puzzle. It describes the <u>constraints</u> that constitute the problem. When S takes the value T corresponds exactly to a solution to the puzzle.

I first label the cells using ~~[illegible]~~ array (= matrix) notation; I am doing this because it will scale up.

Column 1	Column 2	
c_{11}	c_{12}	← row 1 (in fact, the only row)

row column

We shall need 4 atoms to describe what numbers can occur in which cells:

p = 'c_{11} contains the number 1'

q = 'c_{11} contains the number 2'

r = 'c_{12} contains the number 1'

s = 'c_{12} contains the number 2'

Example The truth assignment

p	q	r	s
T	F	T	F

corresponds to the following

1	1

The truth assignment

p	q	r	s
T	F	F	T

corresponds to the following

1	2

We now need to describe the <u>constraints</u> of this Sudoku puzzle using only PL.

<u>Constraints</u>

$p \oplus q$	This is T when C_{11} contains 1 or 2 but not both
$r \oplus s$	This is T when C_{12} contains 1 or 2 but not both
$p \oplus r$	This is T when C_{11} contains 1 or C_{12} contains 1 but not both
$q \oplus s$	This is T when C_{11} contains 2 or C_{12} contains 2 but not both.

Put $S = (p \oplus q) \wedge (r \oplus s) \wedge (p \oplus r) \wedge (q \oplus s)$.

The truth assignments to p, q, r, s that make S T i.e. satisfy S correspond exactly to the solutions to the original Sudoku puzzle.

p	q	r	s	(p⊕q)∧(r⊕s)∧(p⊕r)∧(q⊕s)
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	F
F	T	T	F	T
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

(1)

(2)

Row(1) corresponds to $P = T, S = T$ $q = r = F$

which is the solution $\boxed{1}\boxed{2}$

Row(2) corresponds to $P = F$, $\underbrace{q = T, r = T}$, $s = F$

which is the solution

$\boxed{2}\boxed{1}$