Lecture 4

unary operation

p	$\neg p$
T	F
F	T

binary operations

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	F	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

LEARN THESE

Bonus exercises

Calculate the truth tables of the following:

(1) $((\neg p \leftrightarrow q) \to p) \oplus q$ (2 atoms)

(2) $((\neg p \leftrightarrow q) \wedge r) \vee s$ (4 atoms)

(Just chosen at random)

(1)

p	q	$\neg p$	$\neg p \leftrightarrow q$	$(\neg p \leftrightarrow q) \to p$	$((\neg p \leftrightarrow q) \to p) \oplus q$
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	T	F	T
F	F	T	F	T	T

(2)

p	q	r	s	$\neg p$	$\neg p \leftrightarrow q$	$(\neg p \leftrightarrow q) \wedge r$	$((\neg p \leftrightarrow q) \wedge r) \vee s$
T	T	T	T	F	F	F	T
T	T	T	F	F	F	F	F
T	T	F	T	F	F	F	T
T	T	F	F	F	F	F	F
T	F	T	T	F	T	T	T
T	F	T	F	F	T	T	T
T	F	F	T	F	T	F	T
T	F	F	F	F	T	F	F
F	T	T	T	T	T	T	T
F	T	T	F	T	T	T	T
F	T	F	T	T	T	F	T
F	T	F	F	T	T	F	F
F	F	T	T	T	F	F	T
F	F	T	F	T	F	F	F
F	F	F	T	T	F	F	T
F	F	F	F	T	F	F	F

I checked both of these truth tables using the truth tree generator.

p	q	$((\neg p \leftrightarrow q) \to p) \oplus q$
T	T	F
T	F	T
F	T	T
F	F	T

p	q	r	s	$((\neg p \leftrightarrow q) \land r) \lor s$
T	T	T	T	T
T	T	T	F	F
T	T	F	T	T
T	T	F	F	F
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	F
F	F	T	T	T
F	F	T	F	F
F	F	F	T	T
F	F	F	F	F

Section 1.2

There are two aspects to describing a language:

<u>Syntax</u> or "grammar". This tells us how the symbols of the language are to be arranged.

<u>Semantics</u> or "meaning" tells us how to extract meaning from grammatically correct sequences of symbols.

In this section, we shall describe the <u>syntax</u> of the formal language PL.

Think 'programming language'.

Syntax of PL

- atoms (= atomic statements)

  $p, q, r, \ldots \; p_1, p_2, p_3, \ldots$

- A well-formed formula (or wff) is defined in the following way:

  (WFF1) All atoms are wff.

  (WFF2) If A and B are wff, so too are $(\neg A)$, $(A \wedge B)$, $(A \vee B)$, $(A \oplus B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$

  (WFF3) All wff arise by repeated application of the rules (WFF1) and (WFF2) a finite number of times.

Example We show that

$$(\neg((p \vee q) \wedge r)) \text{ is a wff.}$$

(1) p, q, r are atoms and so wffs.

(2) $(p \vee q)$ is a wff.

(3) $((p \vee q) \vee r)$ is a wff

(4) $(\neg((p \vee q) \vee r))$ is a wff.

Notational convention

(1) Lose outer brackets.

(2) Lose the brackets $(\neg A)$
and write $\neg A$ instead.

Examples

PAY ATTENTION TO BRACKETS!

(1) $\neg p \vee q$
(2) $\neg(p \vee q)$
different.

(3) $(p \wedge q) \vee r$
(4) $p \wedge (q \vee r)$
different.

Parse trees

('parse' means to show the grammar of a sentence)

This is a technique that will be very important when we come to circuit design.

A tree is a data structure.