Lecture 21

Chapter 2

Boolean algebras

Set theory

This topic is needed here and in Chapter 3

A set is a data type.

A set is simply a collection of things.

We use { and } to demarcate what is in the set.

The things in the set are called its elements.

We write $\in$ to mean 'is an element of'

and $\notin$ to mean 'is not an element of'.

Example Define $A = \{1, 2, 3, 4\}$.

This is a set. $1 \in A$, $2 \in A$, $\sqrt{2} \notin A$.

A set is known when its elements are known:

- order is ignored
- repetitions are ignored.

Example $A = \{a, b, c\} = \{c, b, a\} = \{a, a, b, b, c, c\}$

Sets A and B are equal, written $A = B$, when

$x \in A$ if and only if $x \in B$.

Definition $\{\} = \emptyset$, the empty set.

Definition Let A be a set. We say that

B is a subset of A, written $A \subseteq B$, if

B is constructed by choosing only elements of A

$\emptyset \subseteq A$ - choose none.

$A \subseteq A$ - choose all.

The set of all subsets of X is called the powerset of X,

written $P(X)$.

Example $P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$.

Special sets

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, the natural numbers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, the integers

$\mathbb{Q}$ all fraction p/q, the rationals

$\mathbb{R}$ all numbers that can be in decimal notation, the reals

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}.$$

Sets are usually defined by stating the properties their elements are supposed to have. (We shall say more about properties in Chapter 3). Let

$P(x)$ be a statement about the object x.

Example $P(x)$ = 'x is green'

For some things a, $P(a)$ = 'a is green' will be true. Example $P(\text{grass})$ is true but $P(\text{sky})$ is false. Given $P(x)$ we can form the set

$$A = \{a : P(a) \text{ is true}\}$$

(read that)

$$= \{a : P(a)\}$$

Russell's Paradox - see my book

Example

$$\mathbb{P} = \{a : a \in \mathbb{N} \text{ and } a \text{ is prime}\}$$

$$= \{2, 3, 5, 7, \ldots\}.$$