Lecture 12

Revision: Lectures 1 to 11

F17LP Logic and proof 2018

Each question is worth 20 marks

(Throughout this exam paper,
wff is an abreviation for *well formed formula(e)*)

1. (a) Construct truth-tables for $p \wedge q$, $p \vee q$, $p \to q$ and $p \leftrightarrow q$ [4 marks].
   (b) Construct the parse-tree of $(p \leftrightarrow q) \wedge (p \to \neg r)$ [2 marks].
   (c) Construct the truth-table of $(p \leftrightarrow q) \wedge (p \to \neg r)$ [4 marks].
   (d) Construct a wff in disjunctive normal form that has the following truth-table [4 marks].

p	q	r	A
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

   (e) Prove that $p \vee (q \wedge r)$ is logically equivalent to $(p \vee q) \wedge (p \vee r)$ using truth tables. Justify your answer [4 marks].
   (f) What is the *satisfiability problem*? Briefly explain its significance [2 marks].

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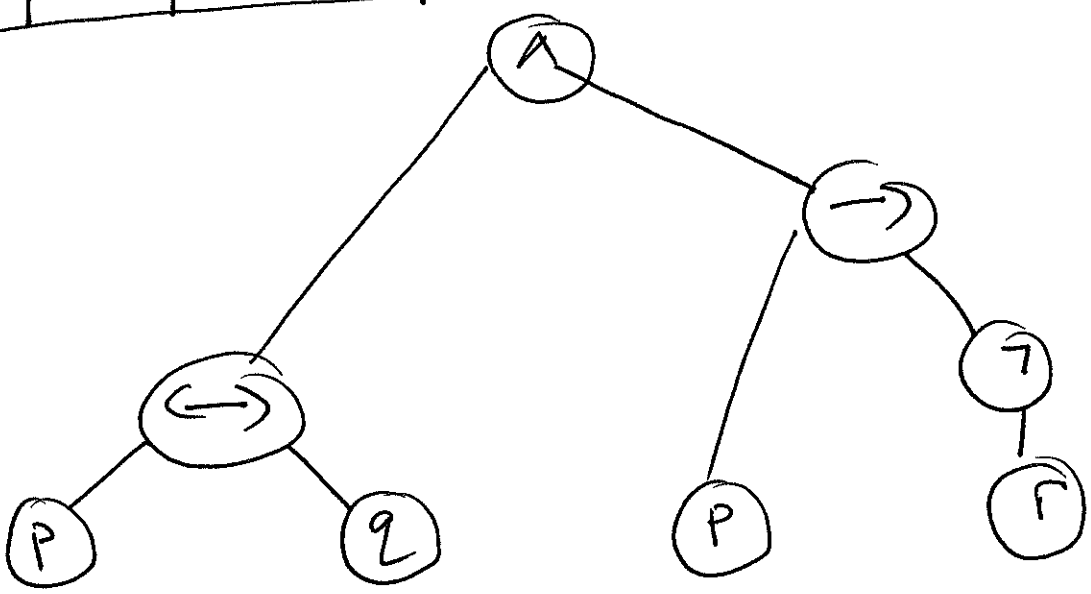
2. (a) Define what is meant by *conjunctive normal form (CNF)* [1 mark]. Given the truth table for a wff A, explain, with reasons, how you would obtain a wff B, in CNF, which was logically equivalent to A [4 marks].

1 (a) You **MUST** learn the truth tables of the basic connectives.

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

(b)



← all atoms here

(c)

p	q	r	$(p\leftrightarrow q)\wedge(p\to\neg r)$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

(d) look at the outputs which are T.

p	q	r	A	
T	T	F	T	1
T	F	F	T	2
F	T	T	T	3
F	F	F	T	4

$$A \equiv (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

(e) We need to draw two truth tables one for $p \vee (q \wedge r)$ and one for $(p \vee q) \wedge (p \vee r)$ OR show that

$(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$ is a tautology. We do the former here.

p	q	r	pV(q∧r)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

p	q	r	(pVq)∧(pVr)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

The truth tables of $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are the same. $\therefore$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

You must write this

(f) The satisfiability problem asks whether a wff is satisfiable or not.

Significance I will discuss this in two lectures time.

2(a) A wff is in CNF if it is a conjunction of one or more blocks where each block is a disjunction of one or more literals.

Given truth table for A.
Compute truth table for $\neg A$
(by simply looking at where A is F).
Write $\neg A$ in DNF using the idea of question 1(d).
Now negate both sides to obtain A in CNF (using double negation and De Morgan's laws).

Example of writing a wff in CNF

We use 1(d).

Look at the outputs when A is F

p	q	r	A	$\neg A$
T	T	T	F	T
T	F	T	F	T
F	T	F	F	T
F	F	T	F	T

Write $\neg A$ in DNF

$$\neg A \equiv (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$$

$$\vee (\neg p \wedge \neg q \wedge r)$$

Now $\neg$ both sides and use double negation

& De Morgan

$$A \equiv (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r)$$
$$\wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$$

CNF

We don't need to write down the truth table for A in CNF but I have done so to show that this method does indeed work.

p	q	r	(¬pV¬qV¬r)Λ(¬pVqV¬r)Λ(pV¬qVr)Λ(pVqV¬r)
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T