Computing braid orbits

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Magaard also wanted to determine the complete list of exceptional simple groups at least for $g = 0$, but if possible also for other small g.

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Note that the tuple on the right is again a generating tuple of G, so we indeed have an action of B_r and orbits.

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The braid orbits, as defined above, classify the loci for the given group G, but only those where the *orbit genus* g_0, that is, the genus of the orbit curve Y/G, is zero (and so $Y = \mathbb{P}^1$).

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This computation was based on the previous work of Breuer, who determined all groups G that can act on compact Riemann surfaces with $2 \leq g < 50$ and also determined all corresponding *types* $(C_1, \ldots, C_r)$.

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This comes from the consideration of the fundamental group of $\hat{Y}$, which is the orbit curve Y with all ramified points removed (punctured).

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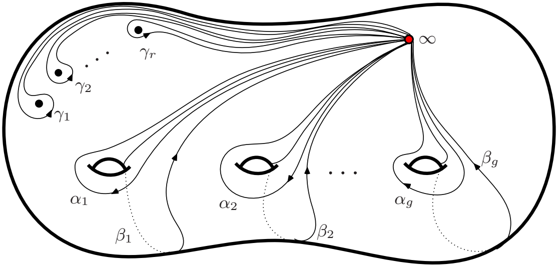

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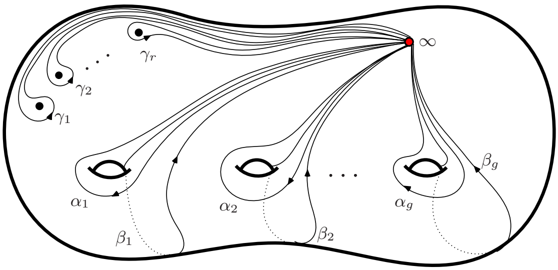

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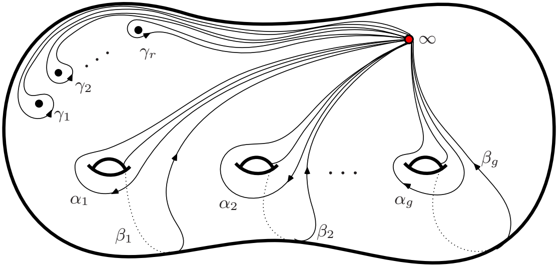

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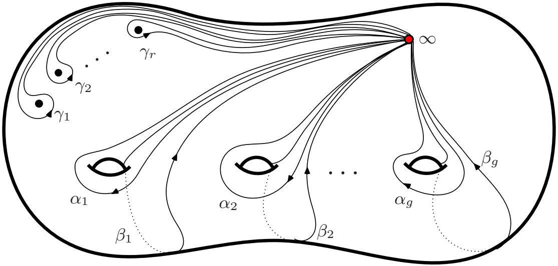

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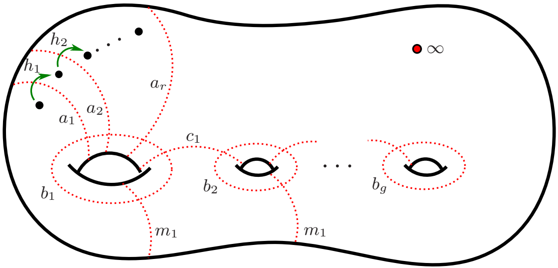

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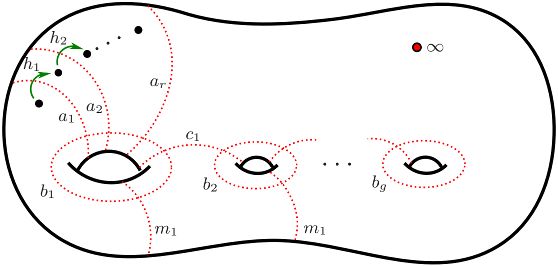

The arrows indicate half-twists (braid twists) and red dotted lines indicate Dehn twists around the suitable loops.

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  - $T_{f_i}(\gamma_j) = (\gamma_{i+1} \cdots \gamma_n \alpha_1^{-1} \beta_1^{-1} \alpha_1) \gamma_j (\gamma_{i+1} \cdots \gamma_n \alpha_1^{-1} \beta_1^{-1} \alpha_1)^{-1}$.

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  - $T_{f_i}(\gamma_j) = (\gamma_{i+1} \cdots \gamma_n \alpha_1^{-1} \beta_1^{-1} \alpha_1) \gamma_j (\gamma_{i+1} \cdots \gamma_n \alpha_1^{-1} \beta_1^{-1} \alpha_1)^{-1}$.
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This action on the standard generators of $\pi_1(\hat{Y}, \infty)$ directly translates to an action of $Mod_{g,r+1}$ on the standard tuples in G, giving us the *mapping class orbits* that classify the arbitrary Hurwitz loci, just like the braid orbits classify the Hurwitz loci with orbit genus zero.

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However, for $g = 3$ and 4, they were compared with the published "hand-made" results. In fact, we corrected a few errors there.
Another interesting feature is that we found many instances of equal loci for different groups.

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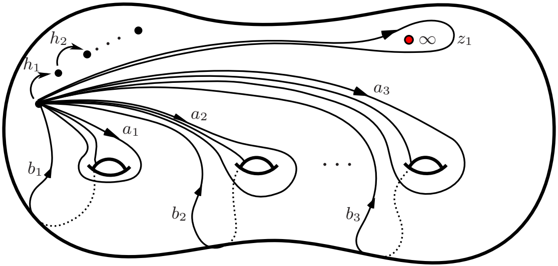

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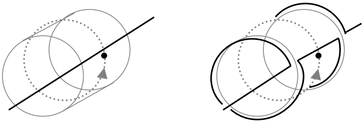

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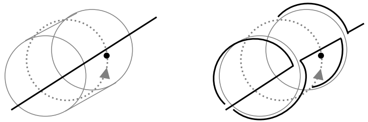

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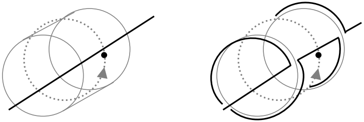

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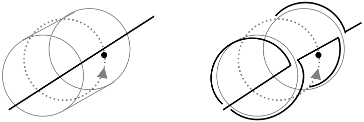

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